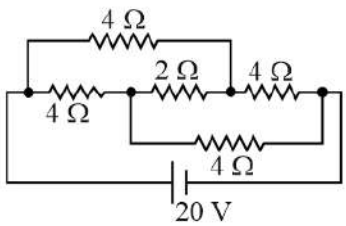
JEE Main Exam 2022 - Session 2

26 Jul 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. In the circuit shown below, find the current supplied by the cell?



A) $2\ \mathrm{A}$

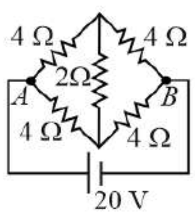
B) $4\ \mathrm{A}$

C) $5\ \mathrm{A}$

D) $10\ \mathrm{A}$

Answer: $5\ \mathrm{A}$

Solution: The circuit can be redrawn as :



We can see that the circuit is a balanced Wheatstone bridge

$\Rightarrow R_{\mathrm{AB}} = 4\ \Omega$

Current through cell $I = \dfrac{V}{R_{\mathrm{AB}}} = \dfrac{20}{4}\ \mathrm{A} = 5\ \mathrm{A}$

Q.2. A charged particle moving in a uniform magnetic field $\vec{B} = 2\hat{\imath} + 3\hat{\jmath}$ has acceleration $\vec{a} = \left(\alpha\hat{\imath} - 4\hat{\jmath}\right)$. The value of α is equal to

A) 6

B) 2

C) $-\frac{8}{3}$

D) $\frac{4}{5}$

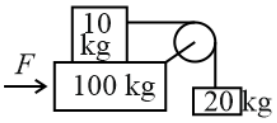
Answer: 6

Solution: The magnetic force on a moving charged particle is given by $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$. Therefore, we know that the magnetic force is perpendicular to the magnetic field. Therefore, the resulting acceleration will also be perpendicular to the magnetic field. As the scalar product of two mutually perpendicular vectors is zero,

$$\Rightarrow \vec{a}\cdot\vec{B}=0$$
$$\Rightarrow \left(\alpha\hat{i}-4\hat{j}\right)\cdot\left(2\hat{i}+3\hat{j}\right)=0$$

$$\Rightarrow 2\alpha-12=0$$
$$\Rightarrow \alpha=6$$

Q.3. Assume all surfaces are friction less. Find value of force required such that $20\ \mathrm{kg}$ block moves with acceleration $2\ \mathrm{m\ s^{-2}}$ upward



A) $2080\ \mathrm{N}$

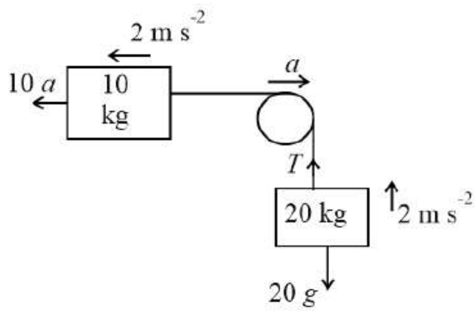
B) $3360\ \mathrm{N}$

C) $2420\ \mathrm{N}$

D) $2820\ \mathrm{N}$

Answer: $3360\ \mathrm{N}$

Solution:



If T is the tension in thread and a be acceleration of $100\,\mathrm{kg}$ block

For the $10\ \mathrm{kg}$ block kept on the $100\ \mathrm{kg}$ block (In the $100\ \mathrm{kg}$ block frame of reference,

$$10a-T=10\times2\ldots(1)$$

For the $20\ \mathrm{kg}$ block in vertical direction,

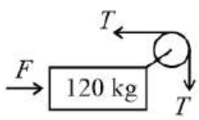
$$T-20g=20\times2$$
$$\Rightarrow T-20\times10=20\times2\ldots(2)$$

Adding the above two equations,

$$\Rightarrow 10a=3\times20+20\times10=260$$

$$a=26\ \mathrm{m\ s^{-2}}, T=240\ \mathrm{N}$$

The applied force F will be responsible for the horizontal acceleration of $100\ \mathrm{kg}$ and $20\ \mathrm{kg}$ block. Therefore,

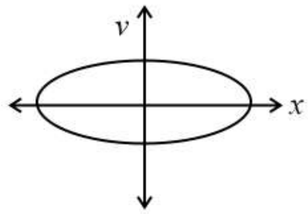


$$F-T=120\ \mathrm{a}$$

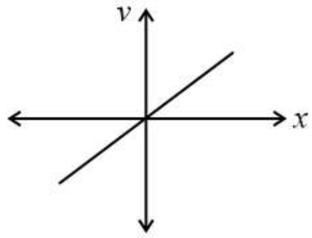
$$\Rightarrow F=3360\ \mathrm{N}$$

Q.4. In S.H.M. $v-x$ graph will be

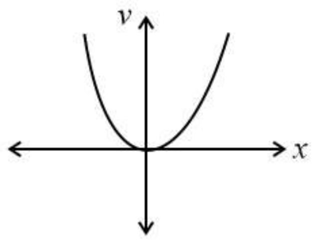
A)



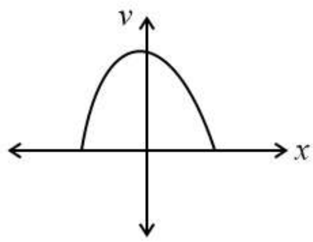
B)



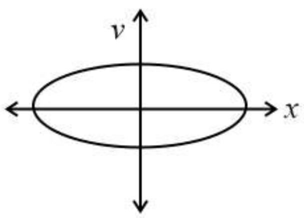
C)



D)



Answer:



Solution: The displacement equation of a particle is given by

$x = A\sin(\omega t+\phi)$

$\Rightarrow \sin(\omega t+\phi) = \frac{x}{A}$

The velocity equation is given by,

$v = \omega A\cos(\omega t+\phi)$

$\Rightarrow \cos(\omega t+\phi) = \frac{v}{\omega A}$

Since, $\sin^2\theta+\cos^2\theta = 1$

$\Rightarrow \frac{x^2}{A^2}+\frac{v^2}{\omega^2 A^2} = 1$

The above equation represents an ellipse.

Q.5. A coil of 200 turns and another coil of 400 turns, they are made from a wire having same length $20\ \mathrm{cm}$. Find the ratio of magnetic field at their centres.

A) $1:2$

B) $2:1$

C) $1:4$

D) $4:1$

Answer: $1:4$

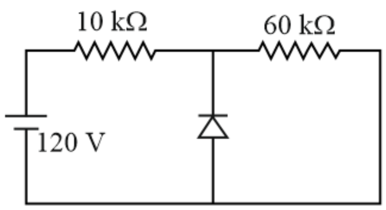
Solution: For a wire bent to form a coil of radius r and number of turns N, its length will be $l = N(2\pi r)$. The magnetic field at the centre of a coil having N turns is given by,

$$B = \frac{N\mu_0 I}{2r} = \frac{N^2\mu_0 I\pi}{2\pi r \times N} = \frac{N^2\pi\mu_0 I}{l}$$

$$\Rightarrow B \propto N^2$$

$$\therefore \frac{B_1}{B_2} = \frac{N_1^2}{N_2^2} = \frac{200^2}{400^2} = \frac{1}{4}$$

Q.6. In the circuit as shown, the potential drop across the diode is $60\ \text{V}$. The current through the diode is _______ mA



A) 5

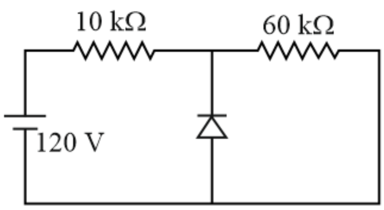
B) 10

C) 15

D) 20

Answer: 5

Solution:



As the diode and the $60\ \text{k}\Omega$ resistance are in parallel, the potential difference across $60\ \text{k}\Omega$ resistance will also be $60\ \text{V}$. Therefore the current through it will be,

$$I_{60} = \frac{60\ \text{V}}{60 \times 10^3 \Omega} = 10^{-3}\ \text{A}$$

The potential difference across $10\ \text{k}\Omega$ resistance will be, $V_{10} = 120 - 60 = 60\ \text{V}$. Therefore,

$$I_{10} = \frac{60\ \text{V}}{10 \times 10^3 \Omega} = 6 \times 10^{-3}\ \text{A}$$

Therefore, the current through diode using Kirchoff's junction law will be

$$I = 6 \times 10^{-3} - 10^{-3} = 5 \times 10^{-3}\text{A} = 5\ \text{mA}$$

Q.7. A drop breaks in 729 smaller identical droplets. If T is the surface tension and R is the radius of bigger drop then change in the surface energy is $n\left(\pi R^2 T\right)$. The value of n is

A) 32

B) 36

C) 728

D) 8

Answer: 32

Solution: The volume of the bigger drop will be, $V=\frac{4}{3}\pi R^3$. If the radius of smaller drops is r, using the conservation of volume,

$$n\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$$
$$\Rightarrow 729\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$$
$$\Rightarrow r=\frac{R}{9}$$

The initial surface energy will be,

$$E_i=\left(4\pi R^2\right)T$$

and final surface energy will be,

$$E_f=n\left(4\pi r^2\right)T=729\times 4\pi\left(\frac{R}{9}\right)^2 T=36\pi R^2 T$$

Therefore, the change in the surface energy will be,

$$\Rightarrow \Delta E=E_f-E_i=32\pi R^2 T$$

$$\Rightarrow n=32$$

Q.8. In a EM wave if amplitude of magnetic field component is $2\times10^{-8}\ \mathrm{T}$, then the value of amplitude of electric field component is ______ $\mathrm{V\ m^{-1}}$.

A) 6

B) 8

C) 4

D) 3

Answer: 6

Solution: The ratio of electric field amplitude to the magnetic field amplitude of an electromagnetic wave is equal to the speed of light in the medium. Therefore,

$$c=\frac{E_0}{B_0}$$
$$\Rightarrow E_0=cB_0$$

$$\Rightarrow E_0=3\times10^8\times2\times10^{-8}$$

$$\Rightarrow E_0=6\ \mathrm{V\ m^{-1}}$$

Q.9. In an LR circuit, if $X_L=R$, then power factor is P_1. In another LCR series circuit if $X_L=X_C$, then power factor is P_2. The value of $\frac{P_1}{P_2}$ is,

A) $1:1$

B) $1:2$

C) $1:\sqrt{2}$

D) $\sqrt{2}:1$

Answer: $1:\sqrt{2}$

Solution: Power factor is given by,$\because$ $P = \frac{R}{Z}$

For first case:

$\Rightarrow P_1 = \frac{R}{Z} = \frac{R}{\sqrt{X_L^2 + R^2}} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}}$

and for second case: $P_2 = \frac{R}{Z} = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{R}{\sqrt{0 + R^2}} = 1$

Therefore,

$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

Q.10. A monkey climbs rope with 4 m s^{-2} acceleration and when it climbs down his acceleration is 5 m s^{-2}. Weight of monkey is 50 kg and maximum tension is 350 N. Then

A) $T = 700 \text{ N}$, when climbs upwards

B) $T = 350\text{N}$, when climbs downwards

C) Rope will break when climbs upward

D) Rope will break when climbs downward

Answer: Rope will break when climbs upward

Solution: Let us assume rope doesn't break, then tension required will be,

$\Rightarrow T_{\text{up}} = m(g + a) = 50 \times 14 = 700 \text{ N}$

But given $T_{\max} = 350$

Clearly, rope will break if the monkey climbs up with acceleration 4 m s^{-2}.

$\Rightarrow T_{\text{down}} = m(g - a) = 50 \times (10 - 5) = 250\text{N}$

Q.11. The wave equation is given as, $y = 2 \times 10^{-8} \sin(kx + \omega t + \theta) \text{ cm}$. Find amplitude?

A) $2 \times 10^{-8} \text{ cm}$

B) $5 \times 10^{-6} \text{ cm}$

C) $4 \times 10^{-6} \text{ cm}$

D) $8 \times 10^{-6} \text{ cm}$

Answer: $2 \times 10^{-8} \text{ cm}$

Solution: Comparing given equation of wave $y = 2 \times 10^{-8} \sin(kx + \omega t + \theta)$ with standard equation of a wave

$y = A \sin(kx + \omega t + \theta)$, we get $A = 2 \times 10^{-8} \text{ cm}$, where A is amplitude.

Q.12. In $YDSE$ experiment, fringe width is $\beta = 12 \text{ cm}$ is given. If the setup is dipped in a medium having refractive index $\mu = \frac{4}{3}$. Find the new fringe width.

A) 6 cm

B) 9 cm

C) 12 cm

D) 16 cm

Answer: 9 cm

Solution: As we know, expression for fringe width is given by, $\beta = \frac{\lambda D}{d}$

For a medium of refractive index μ, we can write,

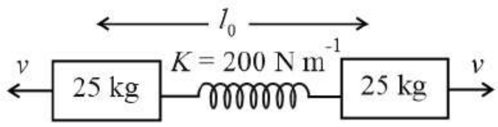
$\beta' = \frac{\lambda D}{\mu d}$

$= \frac{1}{\mu} \times \beta$

$= \frac{3}{4} \times 12$

$\Rightarrow \beta' = 9\ \text{cm}$

Q.13. With spring at its natural length two blocks are given velocity $v = 1\ \text{m s}^{-1}$. The maximum extension in the spring is equal to



A) $5\,\text{cm}$

B) $0.5\ \text{m}$

C) $0.25\ \text{m}$

D) $0.1\ \text{m}$

Answer: $0.5\ \text{m}$

Solution: At maximum elongation, total kinetic energy will be converted into spring potential energy.

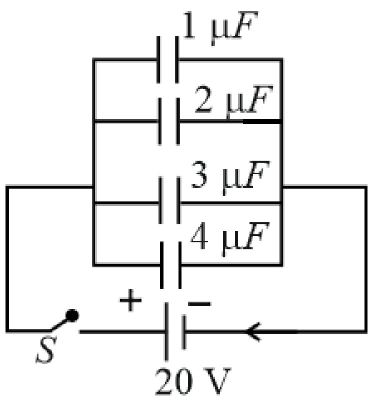
Therefore,

$\frac{1}{2}Kx^2 = 2 \times \left(\frac{1}{2} \times 25 \times v^2\right)$

$\Rightarrow \frac{1}{2} \times 200 \times x^2 = 2 \times \left(\frac{1}{2} \times 25 \times v^2\right)$

$\Rightarrow x = 0.5\ \text{m}$

Q.14. In the circuit as shown, if the switch S is closed, find the total charge flown through the switch.



A) $100\ \mu\text{C}$

B) $50\ \mu\text{C}$

C) $45\ \mu\text{C}$

D) $200\ \mu\text{C}$

Answer: $200\ \mu\text{C}$

Solution: As the given capacitors are in parallel combination, therefore, $C_{\text{eq}} = (1 + 2 + 3 + 4)\ \mu\text{F} = 10\ \mu\text{F}$

Hence, charge flown through the switch $\therefore\ Q = (20\ \text{V}) \times (10\ \mu\text{F}) = 200\ \mu\text{C}$

Q.15. Two projectiles are projected with speed u_1 and u_2 making angles 30° and 45° with the horizontal. Find the value of $\frac{u_1}{u_2}$, if time to reach maximum height is the same.

A) $\sqrt{2}:1$

B) $1:\sqrt{2}$

C) $1:2$

D) $\sqrt{3}:2$

Answer: $\sqrt{2}:1$

Solution: Time of flight is given by, $T=\frac{2u\sin\theta}{g}$

As time of flight is same,

$$\frac{2u_1\sin\theta_1}{g}=\frac{2u_2\sin\theta_2}{g}$$

$$\Rightarrow \frac{u_1}{u_2}=\frac{\sin\theta_2}{\sin\theta_1}=\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}=\sqrt{2}:1$$

Q.16. The decrease in weight of a rocket when it is $32\ \mathrm{km}$ above the surface of earth is,

$(R=6400\ \mathrm{km})$

A) 1%

B) 2%

C) 3%

D) 4%

Answer: 1%

Solution: Acceleration due to gravity at any height is given by,

$\Rightarrow g'=\frac{gR^2}{r^2}$ and weight is given by, $W=mg$

Therefore,

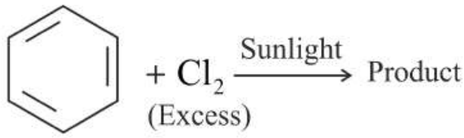
$$\Rightarrow \frac{\Delta W}{W}=\frac{-2\Delta r}{r}$$

$$\Rightarrow \frac{\Delta W}{W}\times 100=\frac{-2\times 32}{6400}\times 100=-1\ \%$$

$\Rightarrow$ decrease in weight 1%

Section B: Chemistry

Q.17.



Number of hydrogen atoms in the product are:

A) 3

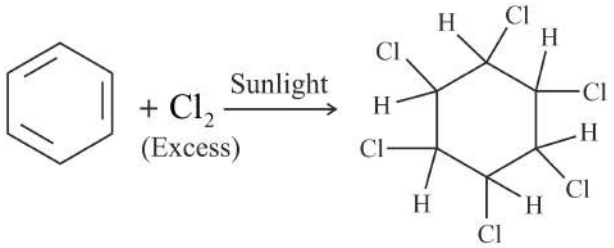
B) 4

C) 6

D) 8

Answer: 6

Solution: In the presence of ultraviolet light (but without a catalyst present), hot benzene will also undergo an addition reaction with chlorine. The ring delocalisation is permanently broken, and a chlorine atom adds on to each carbon atom.

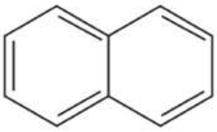


Q.18. Which of the following is not an aromatic compound?

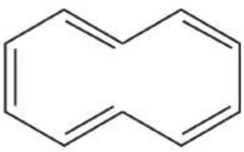
A)

B)

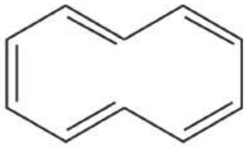
C)



D)

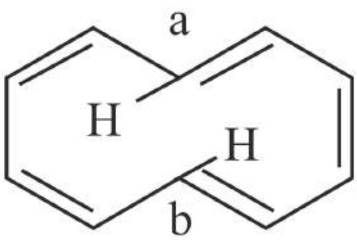


Answer:



Solution: Characteristics of aromatic compounds include:
- Must be Cyclic and planar
- Must have $(4n+2)\pi$ electrons $(n=1,\ 2,\ 3,\ 4,\ \ldots.)$
- Resist Addition but Prefer Substitution.
- Must Possess Resonance Energy.

Due to severe non-bonded interactions between the internal hydrogen (as shown in figure), the ring assumes non-planar geometry in 10-annulene, hence, it is not an aromatic compound.



[10]-Annulene

Q.19. If wavelength of first line of Lyman series of H spectrum is λ and wavelength difference between second transition of Balmer and third transition of Paschen series of line spectrum of H atom is $x\lambda$. Find the value of x.

A) 5

B) 10

C) 15

D) 20

Answer: 5

Solution:

$$\frac{1}{\lambda}=RZ^2\left[\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right]$$

$$\frac{1}{\lambda}=R\left[\frac{1}{1}-\frac{1}{4}\right]=\frac{3R}{4}\Rightarrow\lambda=\frac{4}{3R}$$

$$\frac{1}{\lambda_1}=R\left[\frac{1}{4}-\frac{1}{16}\right]=\frac{12R}{64}\Rightarrow\lambda_1=\frac{16}{3R}$$

$$\frac{1}{\lambda_2}=R\left[\frac{1}{9}-\frac{1}{36}\right]=\frac{27R}{9\times36}\Rightarrow\lambda_2=\frac{36}{3R}$$

$$\lambda_2-\lambda_1=x\lambda=\frac{36}{3R}-\frac{16}{3R}=\frac{20}{3R}$$

$$\frac{20}{3R}=5\times\frac{4}{3R}$$

$$=x\lambda$$

$$x=5$$

Q.20. $\left[Co\left(H_2O\right)_6\right]Cl_2$ and $\left[Co\left(H_2O\right)_6\right]Cl_3$

Find the difference between the spin only magnetic moment of the given compounds.

A) 3.9

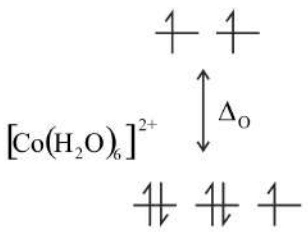
B) 4.9

C) 2.87

D) 1.73

Answer: 3.9

Solution:



Hence, the magnetic moment is given by:

$\mu = \sqrt{n(n+2)}\,BM$, where $n =$number of unpaired electrons.

Hence, the magnetic moment is:

$\mu = \sqrt{3(3+2)}BM = 3.87\,BM$

Hence, the spin-only magnetic moment of $\left[Co(H_2O)_6\right]^{2+}$ is $3.9\ BM$.

$\left[Co(H_2O)_6\right]^{3+}$ is $Co\,(III)$, low spin, 0 unpaired electrons diamagnetic.

Q.21. The velocity of electron is x times the velocity of a neutron. If the wavelength of electron is equal to the wavelength of neutron, find the value of x.

Given: Mass of electron $= 9.1 \times 10^{-31}\ kg$.

Mass of neutron $= 1.6 \times 10^{-27}\ kg$.

(Round off to the nearest integer)

A) 1758

B) 1858

C) 1658

D) 6034

Answer: 1758

Solution: $\lambda_e = \lambda_n$

$\frac{h}{me.ve} = \frac{h}{mn.vn}$

$m_e.\,x.\,v_n = m_n.\,v_n$

$x = \frac{mn}{me} = \frac{1.6\times10^{-27}}{9.1\times10^{-31}}$

$\Rightarrow x = \frac{16}{9} \times 10^3$

$= 1.758 \times 10^3$

$= 1758$

Q.22. Which type of detergents formed by stearic acid and polyethylene glycol?

A) Cation detergent

B) Anionic detergent

C) Soap is formed

D) Non ionic detergent

Answer: Non ionic detergent

Solution: Non-ionic detergents do not contain any ion in their constitution. One such detergent is formed when stearic acid reacts with polyethylene glycol.

$$\underset{\text{Stearic acid}}{CH_3(CH_2)_{16}COOH} + \underset{\text{Polyethyleneglycol}}{HO(CH_2CH_2O)_nCH_2CH_2OH} \xrightarrow{-H_2O} CH_3(CH_2)_{16}COO(CH_2CH_2O)_nCH_2CH_2OH$$

Q.23. The dark purple colour of $KMnO_4$ disappears in the titration with oxalic acid in acidic medium. The overall change of oxidation no. of Mn is

A) 3

B) 1

C) 5

D) 4

Answer: 5

Solution: The overall reaction takes place in the process is

$$2KMnO_4 + 3H_2SO_4 + 5(COOH)_2 \rightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 10CO_2 \uparrow$$

Potassium permanganate is a strong oxidising agent and in the presence of sulfuric acid it acts as a powerful oxidising agent. In acidic medium the oxidising ability of $KMnO_4$ is represented by the following equation.

$$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$$

Q.24. Match the Matrix

	Column I		Column II
A.	$N_2 + 3H_2 \rightarrow 2NH_3$	I.	Pt
B.	$4NH_3 + 5O_2 \rightarrow 4NO + 6H_2O$	II.	Fe
C.	$2SO_2 + O_2 \rightarrow 2SO_3$	III.	V_2O_5

A) A-II, B-III, C-I

B) A-II, B-I, C-III

C) A-III, B-II, C-I

D) A-III, B-I, C-II

Answer: A-II, B-I, C-III

Solution: In Haber's process of producing ammonia, iron is used as a catalyst.

$$N_2 + 3H_2 \xrightarrow{Fe} 2NH_3$$

Catalyst used in Ostwald process is Platinum.

$$4NH_3 + 5O_2 \xrightarrow{Pt} 4NO + 6H_2O$$

Vanadium pentoxide is the catalyst used in the contact process.

$$2SO_2 + O_2 \xrightarrow{V_2O_5} 2SO_3$$

Q.25. A mixture of H_2 and O_2 contains 40% of H_2 by mass. If total pressure is $2.2\ atm$, then calculate the partial pressure of O_2 (in atm).

A) 0.32

B) 0.03

C) 0.19

D) 0.22

Answer: 0.19

Solution: $H_2 \rightarrow 40\ g;\ wt_{H_2} = 2\ gm;\ n_{H_2} = \frac{2}{2} = 1$

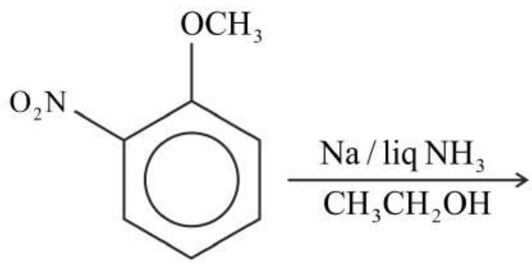
$O_2 \rightarrow 60\ g;\ wt_{O_2} = 3\ gm;\ n_{O_2} = \frac{3}{32}$

$P_{O_2} = X_{O_2} \cdot P_T = \frac{\frac{3}{32}}{\frac{3}{32}+1} \times 2.2 = \frac{\frac{3}{32}}{\frac{35}{32}} \times 2.2$

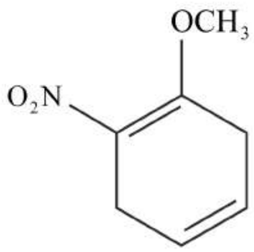
$= \frac{3}{35} \times 2.2$

$\cong 0.19\ atm$

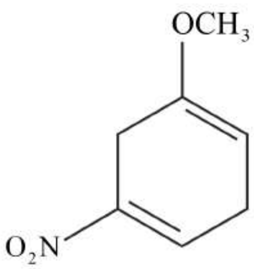
Q.26. The major product formed in the given reaction is:



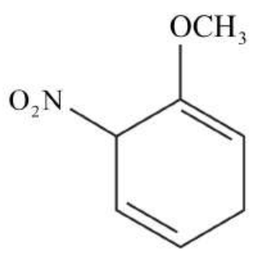
A)



B)

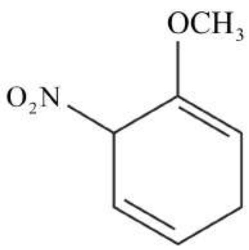


C)

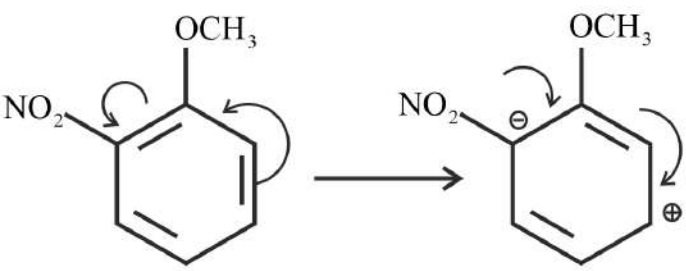


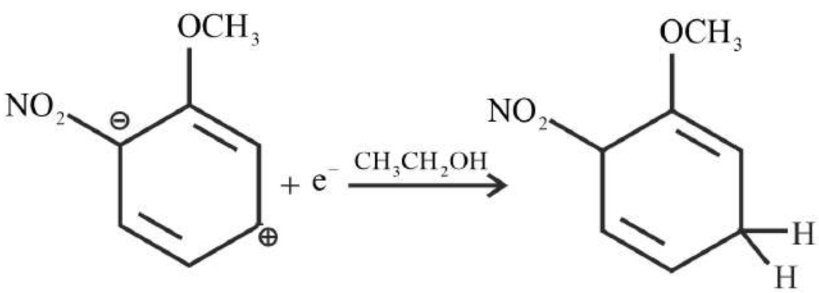
D)

Answer:



Solution: $Na + NH_3 \rightarrow [Na(NH_3)]^{+} + \text{ammoniated electrons}$





Q.27. Which of the following is a non reducing sugar?

A) Sucrose

B) Maltose

C) Lactose

D) Glucose

Answer: Sucrose

Solution: A nonreducing sugar is a carbohydrate that is not oxidized by a weak oxidizing agent (an oxidizing agent that oxidizes aldehydes but not alcohols, such as the Tollens reagent) in basic aqueous solution.

Non-reducing sugars do not have an OH group attached to the anomeric carbon so they cannot reduce other compounds. All monosaccharides such as glucose are reducing sugars. A disaccharide can be a reducing sugar or a non-reducing sugar. Maltose and lactose are reducing sugars, while sucrose is a non-reducing sugar.

Q.28. Chlorophyll is extracted from a leaf. The amount of Mg was $48\ \mathrm{ppm}$. The number of millimoles of Mg in $2\mathrm{L}$ of solution is _____.

[Consider the density of solution is $1\ \mathrm{gm}/\mathrm{mL}$ and molar mass of Mg is $24\ \mathrm{gm}/\mathrm{mol}$.

A) 2

B) 4

C) 6

D) 8

Answer: 4

Solution: Volume of solution $= 2000\ \mathrm{ml}$

Mass of solution $= 2000\ \mathrm{gm}$

$1000\ \mathrm{gm} \longrightarrow 48\ \mathrm{mg}$

$2000\ \mathrm{gm} \ \rightarrow 96\ \mathrm{mg}$

$\mathrm{milli\ \ moles}$ of $\mathrm{Mg} = \frac{96}{24} = 4\ \mathrm{milli\ \ moles}$

Q.29. In summer season, methane reacts with chlorine atom forming chlorine sink, preventing ozone depletion. The products formed in the reaction are:

A) $\dot{\mathrm{C}}\mathrm{H}_3,\ \mathrm{HCl}$

B) $\mathrm{C_2H_6},\ \mathrm{HCl}$

C) $\mathrm{Cl_2},\ \dot{\mathrm{C}}\mathrm{H}_3$

D) $\mathrm{H_2},\ \mathrm{Cl_2}$

Answer: $\dot{\mathrm{C}}\mathrm{H}_3,\ \mathrm{HCl}$

Solution: In summer season, chlorine atoms forming chlorine sinks, preventing much ozone depletion, whereas In winter, special type of clouds called polar stratospheric clouds are formed over Antarctica.

$\dot{\mathrm{C}}\mathrm{l}\,(\mathrm{g}) + \mathrm{CH_4}\,(\mathrm{g}) \rightarrow \dot{\mathrm{C}}\mathrm{H}_3\,(\mathrm{g}) + \mathrm{HCl}\,(\mathrm{g})$

Q.30. Consider the following reactions:

$\mathrm{Cu^{2+}}(\mathrm{aq}) + 2\,\mathrm{Ag}\,(\mathrm{s}) \rightarrow \mathrm{Cu}\,(\mathrm{s}) + 2\,\mathrm{Ag^{+}}(\mathrm{aq})\ \ \mathrm{K_1} = 2 \times 10^{15}$

$\frac{1}{2}\mathrm{Cu}\,(\mathrm{s}) + \mathrm{Ag^{+}}(\mathrm{aq}) \rightarrow \mathrm{Ag}\,(\mathrm{s}) + \frac{1}{2}\mathrm{Cu^{2+}}(\mathrm{aq})\ \ \mathrm{K_2} = ?$

A) 1.14×10^{-7}

B) 2.23×10^{-8}

C) 3.24×10^{-8}

D) 2.56×10^{-7}

Answer: 2.23×10^{-8}

Solution:

$$K_2 = \left(\frac{1}{K_1}\right)^{\frac{1}{2}}$$

$$K_2 = \frac{1}{\sqrt{2\times10^{15}}}$$

$$= \frac{1}{\sqrt{20}\times10^{7}}$$

$$= \frac{10^{-7}}{\sqrt{20}}$$

$$= 0.223 \times 10^{-7}$$

$$= 2.23 \times 10^{-8}$$

Q.31. Which of the following reaction will give borazine?

1. $NH_3 + B_2H_6$
2. $HN_3 + B(OH)_3$
3. $N_2 + B_2H_6$
4. $NH_3 + B(OH)_3$

A) 4

B) 2

C) 3

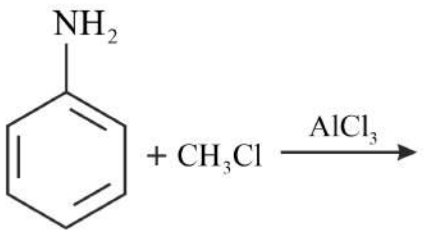
D) 1

Answer: 1

Solution: Reaction of ammonia with diborane gives initially $B_2H_6 . 2NH_3$ which is formulated as $\left[BH_2(NH_3)_2\right]^+ [BH_4]^-$; further heating gives borazine, $B_3N_3H_6$ known as "inorganic benzene" in view of its ring structure with alternate BH and NH groups.

$$3B_2H_6 + 6NH_3 \rightarrow 3\left[BH_2(NH_3)_2\right]^+[BH_4]^- \xrightarrow{\text{Heat}} 2B_3N_3H_6 + 12H_2$$

Q.32.



S1: Ortho and para substituted products are not formed as major product.

S2: Aniline reacts with $AlCl_3$ (Lewis acid-base reaction) and meta substituted product is formed.

Which of the following options is correct?

A) Both S1 and S2 are correct

B) Both S1 and S2 are wrong

C) Only S1 is correct

D) Only S2 is correct

Answer: Only S1 is correct

Solution: Aniline forms salt with the Lewis acid catalyst i.e., $AlCl_3$, which is used in Friedel-Crafts reaction.

Nitrogen of aniline acquires positive charge and hence acts as a strong deactivating group for further reaction.

Hence, no electrophilic substitution reaction takes place.

Q.33. Consider the reaction, $A \rightarrow 2B + C$

It is given that $t_{1/2} = 100 \text{ sec}$ when initial amount of A is 0.5 mol and $t_{1/2} = 50 \text{ sec}$ when initial amount of A is 1.0 mol. Find the order of the reaction.

A) 0

B) 1

C) 2

D) 3

Answer: 2

Solution: $t_{1/2} \propto (a_0)^{1-n}$

$100 \propto \left(\frac{1}{2}\right)^{1-n}$ ... (1)

$50 \propto (1)^{1-n}$ ... (2)

(1) ÷ (2)

$2^1 = 2^{n-1}$

$n - 1 = 1$

$n = 2$

Section C: Mathematics

Q.34. There are ten boys $B_1,\ B_2, \dots, B_{10}$ and five girls $G_1, G_2, \dots G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is _____.

Answer: 1120

Solution: Given, total number of boys is 10 and total number of girls is 5

So total number of selections $= {}^{10}C_3 \cdot {}^{5}C_3 = 1200$

No. of selections in which B_1 & B_2 both are there in group $= {}^{8}C_1 \cdot {}^{5}C_3 = 80$

Required number of selections $= 1200 - 80 = 1120$

Q.35. $\tan\left(2\tan^{-1}\left(\frac{1}{8}\right) + \sec^{-1}\left(\frac{\sqrt{5}}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)\right)$ is equal to

Answer: 2

Solution: Let $T = \tan\left(2\tan^{-1}\left(\frac{1}{8}\right) + \sec^{-1}\left(\frac{\sqrt{5}}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)\right)$

Now, we know $2\left(\tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{5}\right)\right) = 2\tan^{-1}\left(\frac{\frac{1}{8}+\frac{1}{5}}{1-\frac{1}{40}}\right)$

$= 2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right) = \tan^{-1}\frac{3}{4}$

$\Rightarrow T = \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{2}\right)$

$= \tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}\right) = \tan\left(\tan^{-1}2\right) = 2$

Q.36. Let $\{3\}, \{6, 9, 12\}, \{15, 18, 21, 24, 27\}, \cdots$ be any sequence, then the sum of elements in the 11^{th} set of this sequence is _____.

A) 6993

B) 5993

C) 6883

D) 5663

Answer: 6993

Solution: $\{3\}, \{6, 9, 12\}, \{15, 18, 21, 24, 27\}, \cdots$

Now, the number of elements in 11^{th} set will be $= 1 + (10)\,2 = 21$

The total number of elements up to 10^{th} set will be $1 + 3 + \ldots + 19 = 10^2 = 100$

$\therefore$ elements in 11^{th} set $= \{3 \cdot 101, 3 \cdot 102, \ \ldots 3 \cdot 121\}$

Sum of these elements $= 3\,(101 + 102 + \cdots + 121)$

$= 3 \times \frac{21}{2} \times (101 + 121) = 6993$

Q.37. If $\frac{dy}{dx} + 2y\tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$ then maximum value of $y\,(x)$ is

A) $\frac{1}{8}$

B) $-\frac{1}{2}$

C) 1

D) 2

Answer: $\frac{1}{8}$

Solution: $\frac{dy}{dx}+y(2\tan x)=\sin x$ is a linear differential equation

$I.F=e^{\int 2\tan x dx}=e^{2\ln|\sec x|}=\sec^2 x$

The general solution will be

$y\sec^2 x=\int \sin x\sec^2 x dx+C$

$\Rightarrow y\sec^2 x=\sec x+C$

$\because y\left(\frac{\pi}{3}\right)=0\Rightarrow C=-2$

Hence the particular solution is

$y\sec^2 x=\sec x-2$

$\Rightarrow y=\cos x-2\cos^2 x$

$\Rightarrow y=\frac{1}{8}-2\left(\cos x-\frac{1}{4}\right)^2$

So, the maximum value of $y(x)$ is $\frac{1}{8}$

Q.38. Area bounded by the curves $y=1, y=3, y^a=x, (x>0)$ and $x=0$ is $\frac{364}{3}$. Then a is equal to

A) 4

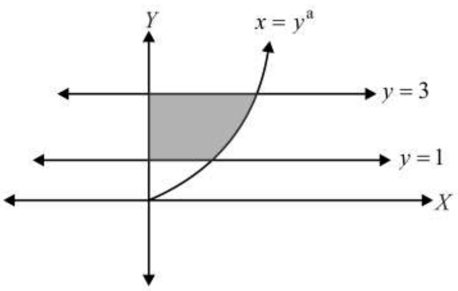
B) 5

C) 6

D) 7

Answer: 5

Solution: Plotting the graph of the given functions, we get the shaded region as the area bounded by the curves.



Given, area of shaded region $=\frac{364}{3}$

$\Rightarrow \int_1^3 y^a dy=\frac{364}{3}$

$\Rightarrow \left[\frac{y^{a+1}}{a+1}\right]_1^3=\frac{364}{3}$

$\Rightarrow \frac{3^{a+1}-1}{a+1}=\frac{364}{3}=\frac{729-1}{6}=\frac{3^6-1}{6}$

$\Rightarrow a=5$

Q.39. If the line $\frac{x+1}{4}=\frac{y-2}{3}=\frac{z-1}{4}$ intersects the plane $x+y-z=0$ at point P. Then distance of P from $Q(2,4,-1)$ is

A) $\sqrt{13}$

B) $\sqrt{17}$

C) $\sqrt{15}$

D) $\sqrt{11}$

Answer: $\sqrt{17}$

Solution: Let $L: \frac{x+1}{4} = \frac{y-2}{3} = \frac{z-1}{4} = \lambda$

and let $P \equiv (4\lambda - 1, 3\lambda + 2, 4\lambda + 1)$ be a point on the line which also lies in $x + y - z = 0$

$\Rightarrow 4\lambda - 1 + 3\lambda + 2 - 4\lambda - 1 = 0$

$\therefore \lambda = 0$

i.e. applying distance formula for $P = (-1, 2, 1)$ and $Q\,(2, 4, -1)$, we get

$PQ = \sqrt{3^2 + 2^2 + (-2)^2} = \sqrt{9 + 4 + 4} = \sqrt{17}$

Q.40. The number of 5 digit numbers whose product of the digits is 36 is

Answer: 180

Solution: Let the five digit number be $abcde$

$\because \quad 36 = 2^2 \times 3^2$

Case I: When exactly one digit is 1

So, number of permutations of $(2, 2, 3, 3, 1)$ will be

$= \frac{5!}{2!2!} = 30$

Case II: When exactly two digits are 1

So, number of permutations of $(4, 3, 3, 1, 1), (6, 2, 3, 1, 1)$ or $(9, 2, 2, 1, 1)$ will be

$= \frac{5!}{2!2!} + \frac{5!}{2!} + \frac{5!}{2!2!} = 120$

Case III: When exactly three digits are 1

So, number of permutations of $(4, 9, 1, 1, 1)$ or $(6, 6, 1, 1, 1)$ will be

$= \frac{5!}{3!} + \frac{5!}{3!2!} = 30$

Hence, the required numbers$= 30 + 120 + 30 = 180$

Q.41. If A is a matrix of order 2×2 and $|A| = -1$ and $|(A + I)\,(\mathrm{adj}\,A + I)| = 4$, then $|\mathrm{Trace}\,(A)|$ is equal to

Answer: 2

Solution: We know that $\mathrm{adj}\,A = |A| A^{-1} = -A^{-1}$

Now, $|(I + A)\,(I + \mathrm{adj}\,A)| = 4$

$\Rightarrow \left|(I + A)\left(I - A^{-1}\right)\right| = 4$

$\Rightarrow |A - A^{-1}| = 4$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = -\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

i.e. $A - A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

So $\begin{bmatrix} a+d & 0 \\ 0 & a+d \end{bmatrix} = 4 \Rightarrow (a + d)^2 = 4$

$\Rightarrow a + d = \pm 2$

Hence, $|\mathrm{Trace}\,A| = 2$

Q.42. Let f be a continuous function such that $f(3x) - f(x) = x$ and $f(8) = 7$, then $f(14)$ equals _____ .

A) 10

B) 9

C) 6

D) 4

Answer: 10

Solution: Given $f(3x) - f(x) = x$

Now

$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$

$f\left(\frac{x}{3}\right) - f\left(\frac{x}{3^2}\right) = \frac{x}{3^2}$

.

.

On adding, we get

$f(x) - \lim_{n\to\infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} + \cdots + \infty\right)$

$f(x) - \lim_{n\to\infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right) = \frac{x}{2}$

$\Rightarrow \quad f(x) - f(0) = \frac{x}{2}$

Given $f(8) = 7$

so $f(8) - f(0) = 4$

i.e. $f(0) = 3$

$\therefore \quad f(x) = \frac{x}{2} + 3$

Hence $f(14) = 10$

Q.43. If $\lim_{n\to\infty} \sum_{k=1}^{n}\left(\frac{2n}{n^2+k^2}\right) = a$ and $f(x) = \sqrt{\frac{(1-\cos x)}{1+\cos x}}$, then $f'\left(\frac{a}{2}\right)$ is equal to:

A) $2+\sqrt{2}$

B) $\sqrt{2}+1$

C) $2-\sqrt{2}$

D) $\sqrt{2}-1$

Answer: $2-\sqrt{2}$

Solution: Given,

$\lim_{n\to\infty} \sum_{k=1}^{n}\left(\frac{2n}{n^2+k^2}\right)=a$

Taking n^2 common from numerator and denominator we get,

$\Rightarrow \lim_{n\to\infty} \sum_{k=1}^{n}\frac{1}{n}\left(\frac{2}{1+\frac{k^2}{n^2}}\right)=a$

$\Rightarrow \int_0^1 \frac{2}{1+x^2}dx=a \Rightarrow 2\left[\tan^{-1}x\right]_0^1=a$

$\Rightarrow a=\frac{\pi}{2}$

Now, $f(x)=\sqrt{\frac{1-\cos x}{1+\cos x}}=\left|\tan\frac{x}{2}\right|$

Differentiating $f(x)$ w.r.t x we get,

$f'(x)=\left(\frac{1}{2}\sec^2\frac{x}{2}\right)$

Now, $f'\left(\frac{a}{2}\right)=\left(\frac{1}{2}\sec^2\frac{x}{2}\right)_{\text{at }x=\frac{\pi}{4}}$

$=\frac{1}{2}\sec^2\frac{\pi}{8}=\frac{\sqrt{2}}{\sqrt{2}+1}=2-\sqrt{2}$

Q.44. A tangent is drawn to $y^2=24x$ at (α,β) which is perpendicular to $2x+2y=7$. Then the equation of normal to hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ at $(\alpha+4,\beta+4)$ is:

A) $2x+5y=100$

B) $2x-5y=100$

C) $2x+5y=10$

D) $2x-5y=10$

Answer: $2x+5y=100$

Solution: Given. $y^2=24x$

Slope of tangent $\left(\frac{dy}{dx}\right)_{(\alpha,\beta)}=\frac{12}{\beta}$

Which is perpendicular to $2x+2y=7$, so slope of tangent will be,

$\frac{12}{\beta}=1\Rightarrow \beta=12$

Now, $\beta^2=24\alpha \Rightarrow \alpha=6$

Now we know that normal to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at (x_1,y_1) is given by, $a^2\frac{x}{x_1}+b^2\frac{y}{y_1}=a^2+b^2$

$\therefore$ Normal to hyperbola $\frac{x^2}{36}-\frac{y^2}{144}=1$ at $(\alpha+4,\beta+4)\equiv(10,16)$ is:

$\frac{36x}{10}+\frac{144y}{16}=180$

$\Rightarrow 2x+5y=100$

Q.45. Given two G.Ps $2,2^2,2^3,\cdots,2^{60}$ and $4,4^2,\cdots,4^n$. If G.M. of $(60+n)$ numbers is $2^{\frac{225}{8}}$, then n equals to

A) 20

B) 48

C) 32

D) 40

Answer: 20

Solution: Given two G.Ps $2, 2^2, 2^3, \cdots, 2^{60}$ and $4, 4^2, \cdots, 4^n$

And geometric mean of all terms $= 2^{\frac{225}{8}}$

Solving L.H.S we get,$= \left(2^{1+2+3+\ldots+60} \times 4^{1+2+3+\ldots+n}\right)^{\frac{1}{60+n}}$

$= \left(2^{30\times61+n(n+1)}\right)^{\frac{1}{60+n}}$

Now comparing with R.H.S we get,

$2^{\frac{30\times61+n(n+1)}{n+60}} = 2^{\frac{225}{8}}$

$\Rightarrow \frac{30\times61+n(n+1)}{n+60} = \frac{225}{8}$

$\Rightarrow 8n^2 - 217n + 1140 = 0 \Rightarrow n = 20$

Q.46. If p, q, r are positive real number such that $\left(p^2 + q^2\right)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ and $x^2 - 2x - 8 = 0$ has one root common then $\frac{q^2+r^2}{p^2}$ is equal to _____ .

A) 272

B) 270

C) 240

D) 260

Answer: 272

Solution: Given, $\left(p^2 + q^2\right)x^2 - 2q(p + r)x + q^2 + r^2 = 0$

On simplifying we get, $(px - q)^2 + (qx - r)^2 = 0$

$\Rightarrow px - q = 0 \ \& \ qx - r = 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q}$

$\Rightarrow \quad x = \frac{q}{p} = \frac{r}{q} = 4$ [because roots of equation $x^2 - 2x - 8 = 0$ are 4 or -2]

As p, q, r are positive, so x must be 4.

Now, $q = 4p$ and $r = 4q = 16p$

So, $\frac{q^2+r^2}{p^2} = \frac{(4p)^2+(4\times4p)^2}{p^2} = 16 + 256 = 272.$

Q.47. If $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 10$, then $265\left(\lim_{h\to0}\frac{h^4+3h^2}{(f(1-h)-f(1))\sin 5h}\right)$ is equal to

Answer: 3

Solution:

$$265\left(\lim_{h\to 0}\frac{h^4+3h^2}{(f(1-h)-f(1))\sin 5h}\right)$$

$$=265\left(\lim_{h\to 0}\frac{\left(h^4+3h^2\right)}{\frac{(f(1-h)-f(1))}{-h}\times\frac{\sin 5h}{5h}\times 5h\times(-h)}\right)$$

$$=265\left(\lim_{h\to 0}\frac{\left(h^2+3\right)}{f'(1^-)\times(-5)}\right)$$

$$=-53\times\frac{3}{f'(1^-)}$$

Given $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-10$

$f'(x)=30x^9-56x^7+30x^5-63x^2+6x$

$\Rightarrow f'\left(1^-\right)=30-56+30-63+6=-53$

$\therefore -53\times\frac{3}{f'(1^-)}=3$