JEE Main Exam 2022 - Session 2

25 Jul 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. If momentum (P), area (A) and time (T) are taken as the fundamental quantities, dimensions of coefficient of viscosity will be

A) $\mathrm{P}^{1}\mathrm{A}^{-1}\mathrm{T}^{0}$

B) $\mathrm{P}^{1}\mathrm{A}^{-2}\mathrm{T}^{1}$

C) $\mathrm{P}^{1}\mathrm{A}^{-1}\mathrm{T}^{-1}$

D) $\mathrm{P}^{-1}\mathrm{A}^{-1}\mathrm{T}^{1}$

Answer: $\mathrm{P}^{1}\mathrm{A}^{-1}\mathrm{T}^{0}$

Solution: Viscous force is given by the formula, $F=-\eta A\frac{\mathrm{d}v}{\mathrm{d}z}$

$$\Rightarrow \eta=-\frac{F}{A\left(\frac{\mathrm{d}v}{\mathrm{d}z}\right)}=-\frac{\frac{\Delta P}{\Delta t}}{A\left(\frac{\mathrm{d}v}{\mathrm{d}z}\right)}$$

Since,

$$\left[\frac{\mathrm{d}v}{\mathrm{d}z}\right]=\frac{\mathrm{LT}^{-1}}{\mathrm{L}}=\mathrm{T}^{-1}$$

$$\Rightarrow [\eta]=\mathrm{P}^{1}\mathrm{A}^{-1}\mathrm{T}^{0}$$

Q.2. A particle moves on a circular path of radius R and the angle subtended by its initial position and the final position at the centre is 135°, find its displacement.

A) $R\left(\sqrt{2+\sqrt{2}}\right)$

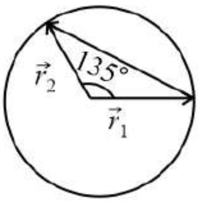
B) $R\left(2+\sqrt{2}\right)$

C) $\sqrt{2}R$

D) $\sqrt{3}R$

Answer: $R\left(\sqrt{2+\sqrt{2}}\right)$

Solution:



Displacement of the particle will be,

$$\vec{s}=\vec{r_2}-\vec{r_1}$$

$$\Rightarrow \left|\vec{s}\right|=\sqrt{r_2{}^2+r_1{}^2-2r_2r_1\cos 135^{\circ}}$$

$$\Rightarrow \left|\vec{s}\right|=\sqrt{R^2+R^2-2R^2\left(-\frac{1}{\sqrt{2}}\right)}$$

$$\Rightarrow \left|\vec{s}\right|=R\left(\sqrt{2+\sqrt{2}}\right)$$

Q.3. A solid cylinder and a solid sphere is rolled from inclined plane with initial velocity 0, then ratio of their velocity at down the incline

A) $\sqrt{\frac{14}{15}}$

B) $\sqrt{\frac{15}{14}}$

C) $\frac{2}{5}$

D) $\frac{5}{2}$

Answer: $\sqrt{\frac{14}{15}}$

Solution: During pure rolling, the work done by the friction is zero. Therefore, we can apply conservation of mechanical energy.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$\Rightarrow mgh = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2$$

$$\Rightarrow v = \sqrt{\frac{2gh}{1+\frac{I}{mR^2}}}$$

For a solid cylinder $\frac{I}{mR^2} = \frac{1}{2}$ and for a solid sphere, $\frac{I}{mR^2} = \frac{2}{5}$

$$\frac{v_{\text{Cyl}}}{v_{\text{Sph}}} = \sqrt{\frac{1+\frac{2}{5}}{1+\frac{1}{2}}} = \sqrt{\frac{14}{15}}$$

Q.4. For a given object moving with some velocity, the stopping distance is $27\ \mathrm{m}$. If the velocity is reduced to $\left(\frac{1}{3}\right)^{\text{rd}}$ of its initial value, then the stopping distance in metre will be

A) $3\ \mathrm{m}$

B) $9\ \mathrm{m}$

C) $6\ \mathrm{m}$

D) $12\ \mathrm{m}$

Answer: $3\ \mathrm{m}$

Solution: In both cases, the final velocity is zero. Using equation of motion for constant acceleration

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0^2 - u^2 = -2a(27) \quad \ldots(1)$$

$$\Rightarrow 0^2 - \left(\frac{u}{3}\right)^2 = -2as' \quad \ldots(2)$$

From (1) and (2),

$$2a(27) = 9\left(2as'\right)$$

$$\Rightarrow s' = 3\ \mathrm{m}$$

Q.5. The ratio of $F_1 : F_2 = 1 : X$. Then find the value of X, when net force is zero.

A) 3

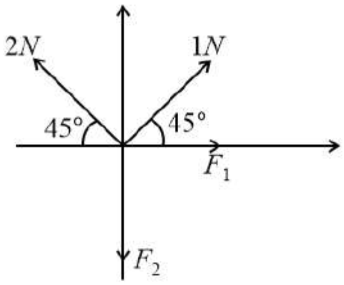
B) 4

C) 5

D) 6

Answer: 3

Solution:



As the net force is zero:

$\Rightarrow \sum F_x = 0$ & $\sum F_y = 0$

$F_1 - (2\cos 45^\circ - 1\cos 45^\circ) = 0$

$\Rightarrow F_1 = \frac{1}{\sqrt{2}}$ N

$F_2 - (2\sin 45^\circ + \sin 45^\circ) = 0$

$\Rightarrow F_2 = \frac{3}{\sqrt{2}}$ N

Therefore,

$F_1 : F_2 = 1 : 3$
$\Rightarrow X = 3$

Q.6. Two positively charged particle A and B having same kinetic energy move inside a transverse uniform magnetic field. The ratio of radius of path of charge A to that of charge B is $\frac{3}{5}$, while mass of particle A is $\frac{4}{9}$ times that of particle B. The ratio of charge on particle A to that B is

A) $\frac{9}{10}$

B) $\frac{10}{9}$

C) $\frac{2}{5}$

D) $\frac{5}{3}$

Answer: $\frac{10}{9}$

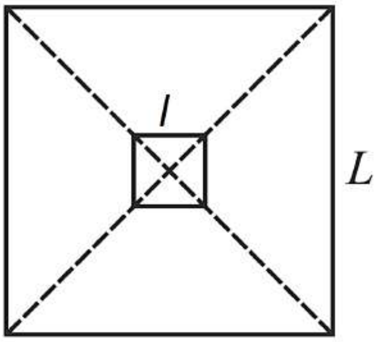
Solution: Radius of the circular path is given by, $r = \frac{mv}{qB} = \frac{\sqrt{2m(KE)}}{qB}$

$\Rightarrow \frac{r_A}{r_B} = \sqrt{\frac{m_A}{m_B}} \times \frac{q_B}{q_A}$

$\Rightarrow \frac{q_A}{q_B} = \sqrt{\frac{m_A}{m_B}} \times \frac{r_B}{r_A}$

$= \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$

Q.7. A small square loop of wire of side l is placed inside a large square loop of wire of side $L(\gg l)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is



A) $\frac{2\sqrt{2}\mu_0 L}{\pi l}$

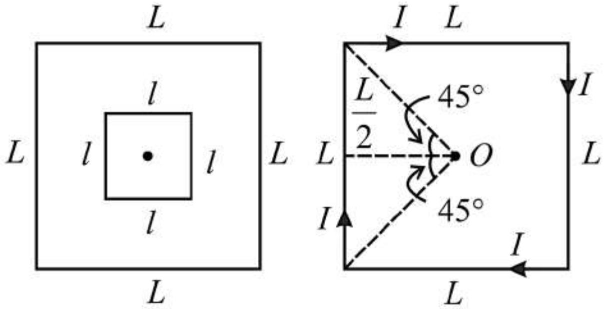
B) $\frac{2\sqrt{2}\mu_0 L^2}{\pi l}$

C) $\frac{2\sqrt{2}\mu_0 l}{\pi L}$

D) $\frac{2\sqrt{2}\mu_0 l^2}{\pi L}$

Answer: $\frac{2\sqrt{2}\mu_0 l^2}{\pi L}$

Solution:



Let a current, I pass through the square loop of side, L. The magnetic field at the centre O,

$B = 4 \times \text{Magnetic field due to each side}$

$\Rightarrow B = 4 \times \frac{\mu_0}{4\pi} \cdot \frac{I}{L/2}(\sin 45° + \sin 45°)$

$\Rightarrow B = \frac{2\mu_0 I}{\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{2\sqrt{2}\mu_0 I}{\pi L}$

Magnetic flux linked with the small square loop,

$\phi = BA = Bl^2 = \frac{2\sqrt{2}\mu_0 I l^2}{\pi L}$

Mutual inductance of the coil,

$M = \frac{\phi}{I} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}.$

Q.8. A body of mass $0.5\ \text{kg}$ has velocity $3x^2 + 5$. What is the work done in moving it from $x = 0$ to $x = 2\ \text{m}$?

A) $60\ \text{J}$

B) $64\ \text{J}$

C) $66 \mathrm{~J}$

D) $72 \mathrm{~J}$

Answer: $66 \mathrm{~J}$

Solution: Given: $v=3 x^2+5$

$v_i=3(0)^2+5=5 \mathrm{~m} \mathrm{~s}^{-1}$

and $v_f=3(2)^2+5=17 \mathrm{~m} \mathrm{~s}^{-1}$

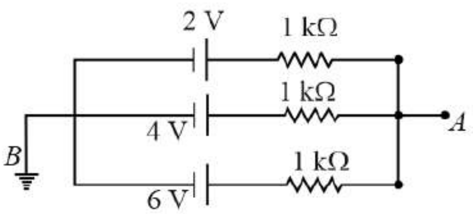
From work-energy theorem,

$\Rightarrow \Delta K E=\frac{1}{2} m\left(v_f^2-v_i^2\right)$

$\Rightarrow \Delta K E=\frac{1}{2} \times \frac{1}{2}\left(17^2-5^2\right)$

$\Rightarrow \Delta K E=W=66 \mathrm{~J}$

Q.9. For the circuit shown below potential difference across point A and B is



A) $4 \mathrm{~V}$

B) $3 \mathrm{~V}$

C) $2 \mathrm{~V}$

D) $1 \mathrm{~V}$

Answer: $4 \mathrm{~V}$

Solution:

Equivalent EMF for parallel combination of cells with resistance: $E=\frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}+\frac{\varepsilon_3}{r_3}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$

Therefore, $E_{A B}=\frac{\frac{2}{1000}+\frac{4}{1000}+\frac{6}{1000}}{\frac{1}{1000}+\frac{1}{1000}+\frac{1}{1000}}=4 \mathrm{~V}$

Q.10. Work function of a photosensitive material is $4.0 \mathrm{~eV}$. The longest wavelength of light that can cause photoemission from the substance is

A) $310 \mathrm{~nm}$

B) $470 \mathrm{~nm}$

C) $3100 \mathrm{~nm}$

D) $955 \mathrm{~nm}$

Answer: $310 \mathrm{~nm}$

Solution: Given: work function, $\phi=4 \mathrm{~eV}$

At largest wavelength energy of photon capable to do photoelectric emission is equal to $4 \mathrm{~eV}$.

$\lambda_{\text {photon }}=\frac{1240}{E(\text { in eV })} \mathrm{nm}$

$=\frac{1240}{4}=310 \mathrm{~nm}$

$\Rightarrow \lambda_{\text {photon }}=310 \mathrm{~nm}$

Q.11. 8 resistance of length l, diameter d are connected in parallel, equivalent resistance is found to be R. If R is also the resistance of wire length $2l$, then its diameter is $\ldots\ldots d$.

A) 4

B) 5

C) 6

D) 7

Answer: 4

Solution: For parallel combination, $R=\left(\frac{1}{8}\right)\times r=\frac{1}{8}\frac{\rho l}{\left(\frac{\pi d^2}{4}\right)}=\frac{1}{2}\frac{\rho l}{\pi d^2}$

and it is also given that for a wire of $2l$, resistance is same.

Therefore, $R=\frac{\rho(2l)}{\frac{\pi(d')^2}{4}}=8\frac{\rho l}{\pi(d')^2}$

Comparing both values, we get,

$(d')^2=16d^2$

$\Rightarrow d'=4d$

Q.12. The RMS value of current flowing through capacitor is $6\ \mathrm{A}$ and voltage across it is $230\ \mathrm{V}$, If angular frequency is $60\ \mathrm{rad\ s^{-1}}$, then capacitance of capacitor

A) $435\ \mu\mathrm{F}$

B) $576\ \mu\mathrm{F}$

C) $176\ \mu\mathrm{F}$

D) $783\ \mu\mathrm{F}$

Answer: $435\ \mu\mathrm{F}$

Solution: $I_{\mathrm{rms}}=6A;\ V_{\mathrm{rms}}=230\ \mathrm{V};\ \omega=60\ \mathrm{rad\ s^{-1}}$

As we know, capacitive reactance can be written as, $X_C=\frac{1}{\omega C}$

Therefore,

$\frac{1}{\omega C}=\frac{V_{\mathrm{rms}}}{I_{\mathrm{rms}}}=\frac{230}{6}$

$\Rightarrow C=\frac{6}{60\times 230}$

$=\frac{100}{23}\times 10^{-4}$

$=43.5\times 10^{-5}\ \mathrm{F}$

$=435\ \mu\mathrm{F}$

Q.13. Linear momentum of an electron in an orbit is given by,

A) $\frac{nh}{2\pi r}$

B) $\frac{2\pi}{hr}$

C) $\frac{n\pi}{hr}$

D) $\frac{hr}{2n\pi}$

Answer: $\frac{nh}{2\pi r}$

Solution: According to Bohr's postulates, angular momentum of an electron in an orbit is given by,

$L = mvr = \frac{nh}{2\pi}$

$\Rightarrow p = \frac{L}{r} = \frac{nh}{2\pi r}$

Q.14. In a YDSE setup, red light of wavelength 700 nm is used. If the slit separation is 4 mm and the screen is placed 2 m away, then find the width of fringe.

A) 0.2 mm

B) 0.75 mm

C) 1 mm

D) 0.35 mm

Answer: 0.35 mm

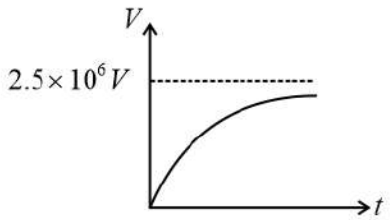
Solution: The fringe width in a YDSE is given by $\beta = \frac{\lambda D}{d}$.

$\Rightarrow \beta = \frac{700 \times 10^{-9} \times 2}{4 \times 10^{-3}} = \frac{14}{4} \times 10^{-4}$

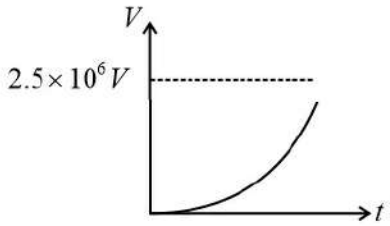
$\Rightarrow \beta = 0.35 \text{ mm}$

Q.15. A $2 \mu\text{F}$ capacitor is being charged from 0 to a maximum of 5 C. Voltage across the capacitor varies as

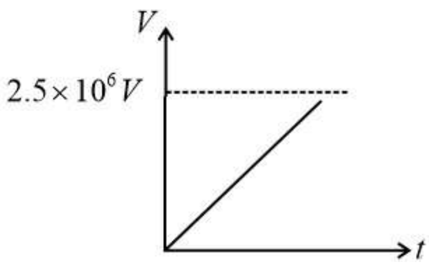
A)



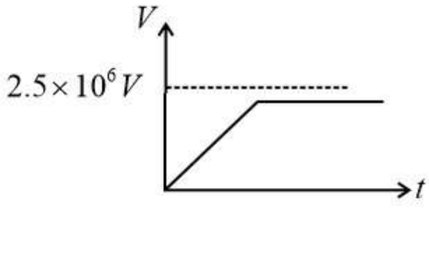
B)



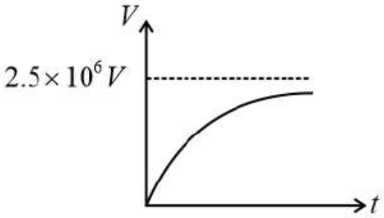
C)



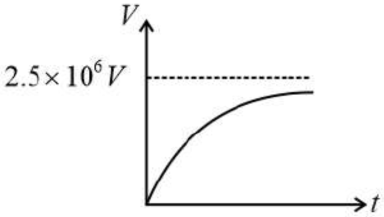
D)



Answer:



Solution:



Maximum voltage on the capacitor is given by,

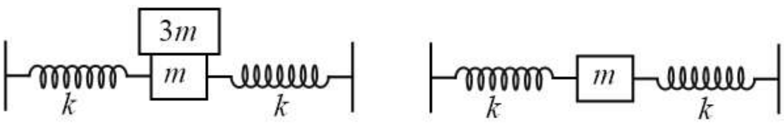
$$\Rightarrow V_{\max} = \frac{Q_{\max}}{C} = \frac{5}{2 \times 10^{-6}} \text{ V} = 2.5 \times 10^6 \text{ V}$$

Now the voltage across capacitor follow the relation

$$\Rightarrow V = V_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

Graph given in option A satisfies the relation

Q.16. For the system shown in figure, ratio of time period of oscillation $\frac{T_1}{T_2}$ is (T_1 is the time period of oscillation of left combination & T_2 for the right combination. Assume no relative slipping between blocks)



A) $\frac{2}{1}$

B) $\frac{1}{2}$

C) $\frac{4}{1}$

D) $\frac{1}{4}$

Answer: $\frac{2}{1}$

Solution: The time period of spring block system is given by $T = 2\pi\sqrt{\frac{m}{k}}$.

In both the cases springs are connected in parallel, therefore , $k_{eq} = 2k$

In case 1: $T_1 = 2\pi\sqrt{\frac{(m+3m)}{keq}} = 2\pi\sqrt{\frac{(4m)}{2k}}$

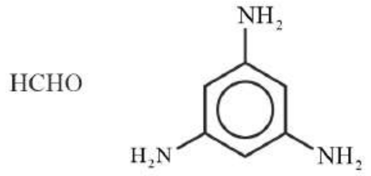
In case 2: $T_2 = 2\pi\sqrt{\frac{(m)}{2k}}$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{4}{1}} = \frac{2}{1}$$

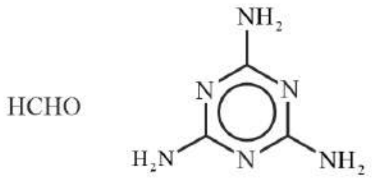
Section B: Chemistry

Q.17. Melamine resin is made up of

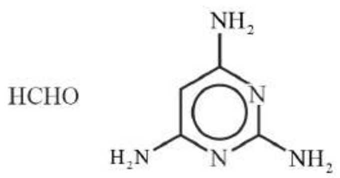
A)



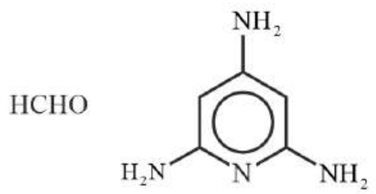
B)



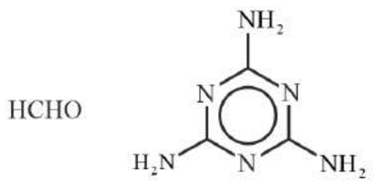
C)



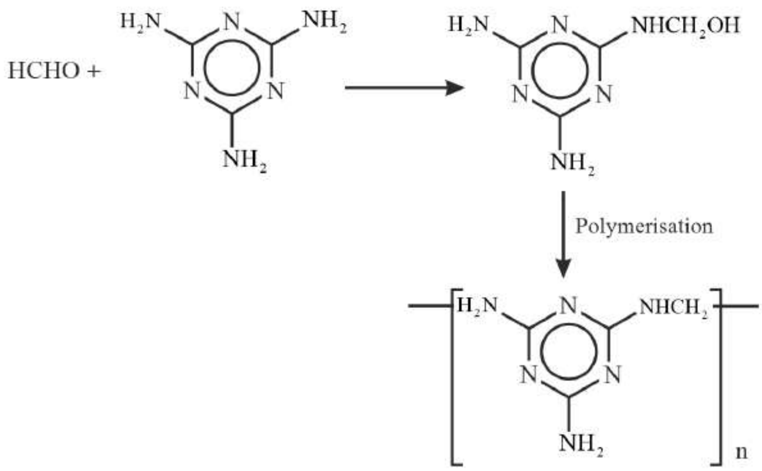
D)



Answer:



Solution: Melamine is a nitrogen-based compound used by many manufacturers to create a number of products, especially plastic dish ware. It's also used in plastic products, Dry-erase boards.



Q.18. Which of the following compounds is not a photochemical smog?

A) NO

B) NO_2

C) SO_2

D) PAN

Answer: SO_2

Solution: The major components of photochemical smog are nitrogen oxides, ozone (O_3), PAN (peroxyacetylnitrate), and chemical compounds that contain the-CHO group (aldehydes). PAN and aldehydes can cause eye irritation and plant damage if their concentrations are sufficiently high.

Classical smog occurs in a cool, humid climate. Its components include smoke, fog and sulphur dioxide. It is reducing in nature.

Q.19. Denatured protein contains which type of structure?

A) Primary

B) Secondary

C) Tertiary

D) Quarternary

Answer: Primary

Solution: Protein structures are made by condensation of amino acids forming peptide bonds. The sequence of amino acids in a protein is called its primary structure. The secondary structure is determined by the dihedral angles of the peptide bonds, the tertiary structure by the folding of protein chains in space.

Denaturation involves the breaking of many of the weak linkages, or bonds (e.g., hydrogen bonds), within a protein molecule that are responsible for the highly ordered structure of the protein in its natural (native) state.

During denaturation 2° and 3° structures are destroyed but 1° structure remains intact

Q.20. The increasing order of the density of alkali metals is

A) $Li < K < Na < Rb < Cs$

B) $Li < Na < K < Rb < Cs$

C) $Cs < Rb < Na < K < Li$

D) $Cs < Rb < K < Na < Li$

E) $Li < Na < Rb < K < Cs$

Answer: $Li < K < Na < Rb < Cs$

Solution: As we move down the group from Li to Cs, both the atomic size and atomic mass increase. But the increase in atomic mass more than compensates the bigger atomic size. As a result, the densities of alkali metals increase from Li to Cs. Potassium is, however, lighter than sodium.

Hence, the increasing order of density is as given below:

$Li < K < Na < Rb < Cs$

Q.21. IUPAC name of $[Rn]5f^{14}6d^{1}7s^{2}$ is:

A) Unnilunium

B) Unnilbium

C) Unniltrium

D) Unnilseptium

Answer: Unniltrium

Solution: IUPAC nomenclature has been recommended for all the elements with $Z > 100$. It was decided by IUPAC that the names of elements beyond atomic number 100 should use Latin words for their numbers. The names of these elements are derived from their numerical roots.

Numerical	0	1	2	3	4	5	6	7	8	9
roots	nil	un	bi	tri	quad	pent	hex	sept	oct	en

The atomic mass of given molecule $[Rn]5f^{14}6d^{1}7s^{2} = 86 + 14 + 1 + 2 = 103$

Name of element with this atomic number $= \text{un} + \text{nil} + \text{tri} + \text{ium} = \text{unniltrium}$

Q.22. The pK_a of a weak acid , HA , is 4.80 . The pK_b of a weak base , BOH is 4.78 . The pH of an aqueous solution of the corresponding salt BA will Be?

A) 9.22

B) 9.58

C) 4.79

D) 7.01

Answer: 7.01

Solution: Salt of weak acid & weak base

Given that pK_a = 4.8 and pK_b =4.78

$\therefore \text{pH} = 7 + 1/2\left(\text{p}K_a - \text{p}K_b\right)$

$= 7 + 1/2\,(4 \cdot 80 - 4 \cdot 78) = 7 \cdot 01$

Q.23. Drugs that bind to receptor site and inhibit the natural function are known as

A) antagonists.

B) agonists.

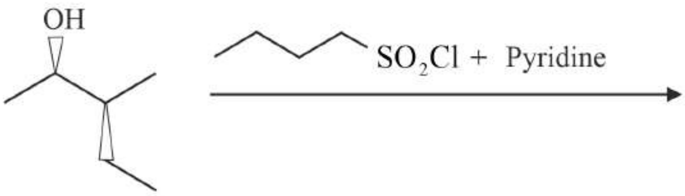
C) allosteric.

D) lead compounds.

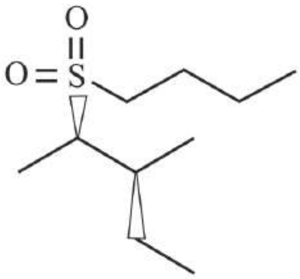
Answer: antagonists.

Solution: Drugs that bind to receptor site and inhibit the natural functions are known as antagonists. Agonist are type of drugs that activate the natural messenger by switching on the receptor, useful when lack of chemical messenger. If the drug binds to a site other than active site in an enzyme, it is known as allosteric site.

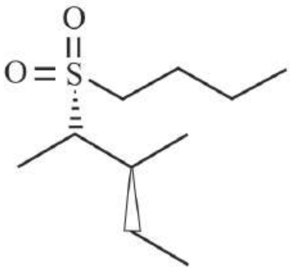
Q.24.



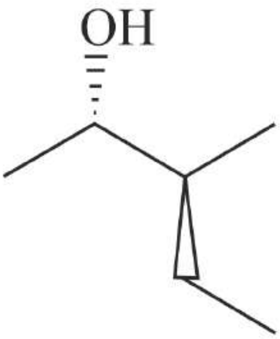
A)



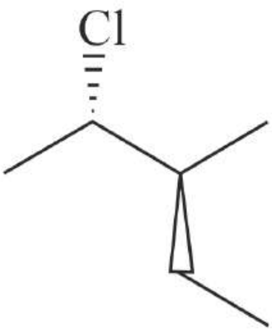
B)



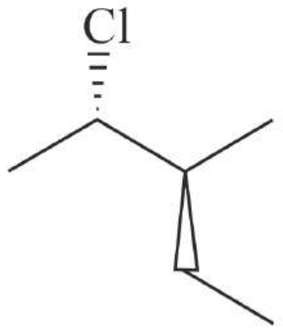
C)



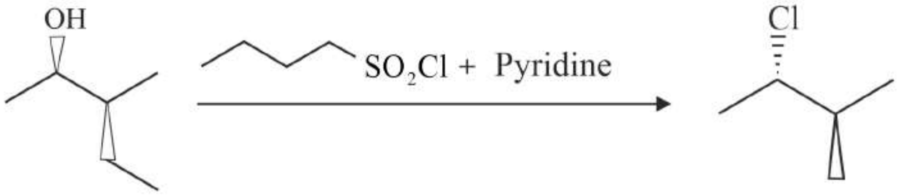
D)



Answer:



Solution: This reaction is an example of Darzen's reaction. It follows S_N2 mechanism. Hence, inversion product is the major product.



Q.25. $SO_2Cl_2 + 2H_2O \rightarrow 2HCl + H_2SO_4$

If 16 moles of $NaOH$ are required to neutralise acid formed, how many moles of SO_2Cl_2 were present initially?

A) 16

B) 8

C) 4

D) 2

Answer: 4

Solution: $SO_2Cl_2 + 2H_2O \rightarrow 2HCl + H_2SO_4$

Let x moles of SO_2Cl_2 are required,

Moles of H^+ formed $= 4x$

Moles of OH^- from 16 moles of $NaOH = 16$

$4x = 16$ (for complete neutralisation)

$x = 4$

Q.26. Which of the following compounds absorb shortest wavelength of light?

A) $[Co(NH_3)_6]^{3+}$

B) $[Co(NH_3)_5H_2O]^{3+}$

C) $[Co(NH_3)_5Cl]^{3+}$

D) $[Co(CN)_6]^{3-}$

Answer: $[Co(CN)_6]^{3-}$

Solution: The complexes with strong field ligands absorbs more energetic light.

$E = hc/\lambda$

The photon energy is inversely proportional to the wavelength of the electromagnetic wave. The shorter the wavelength, the more energetic is the photon

The ligand strength order is

$Cl^- < H_2O < NH_3 < CN^-$

So, among the given options $[Co(CN)_6]^{3-}$ has the strongest ligand so, it will have maximum splitting energy and will absorb light with maximum energy or shortest wavelength.

Q.27. S_1 : Acrolein can be obtained from glycerol by treatment with $KHSO_4$.

S_2 : Acrolein has a fruity smell and presence of glycerol can be identified with acrolein.

A) S_1 and S_2 both are correct

B) S_1 and S_2 both are incorrect

C) S_1 is correct S_2 is incorrect

D) S_1 is incorrect S_2 is correct

Answer: S_1 is correct S_2 is incorrect

Solution:

Acrolein
(Pungent odour)

It is a colorless liquid with a piercing, acrid smell. The smell of burnt fat (as when cooking oil is heated to its smoke point) is caused by glycerol in the burning fat breaking down into acrolein. It does not have fruity smell so, statement 2 is incorrect.

Q.28. How many of the following compounds is/are diamagnetic?

$N_2, N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}$

A) 3

B) 4

C) 5

D) 2

Answer: 2

Solution: The molecules with unpaired electrons are paramagnetic and molecules with all electrons paired are diamagnetic. In general molecules with an even number of electrons (except 10 and 16 electrons) are diamagnetic

N_2 $14e^-$ diamagnetic

N_2^+ $13e^-$ paramagnetic

N_2^- $15e^-$ paramagnetic

N_2^{2-} $16e^-$ paramagnetic

O_2 $16e^-$ paramagnetic

O_2^+ $15e^-$ paramagnetic

O_2^- $17e^-$ paramagnetic

O_2^{2-} $18e^-$ diamagnetic

Q.29. Number of sp^3 carbon in C_4H_5N

A) 2

B) 3

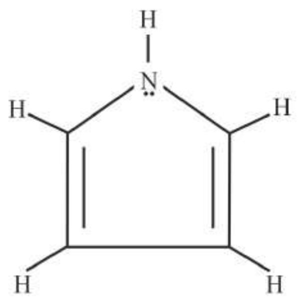
C) 0

D) 1

Answer: 0

Solution: Pyrrole is a heterocyclic aromatic organic compound, a five-membered ring with the chemical formula C_4H_5N. It is a planar molecule and all the carbon atoms are in sp^2 hybridisation. So, number of sp^3 hybridised carbon is 0.

$$DOU = 4 - \frac{5}{2} + \frac{1}{2} + 1 = 3$$



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Q.30. In the metallurgy of copper which one gets removed in form of slag:

A) CaO

B) FeO

C) NiO

D) ZnO

Answer: FeO

Solution: Copper ores contain iron as the main impurity. Iron react with silica and is removed as slag. Therefore, the slag formed in the blast furnace is $FeSiO_3$.

The reaction that takes place in the blast furnace is as follows:

$FeO + SiO_2 \rightarrow FeSiO_3$(slag)

Q.31. What happens to viscous force with changes in temperature (T) and area (A)?

A) Increases with increase in temperature as well as area

B) Decreases with decrease in temperature as well as area

C) Increases with increase in temperature and decreases with increase in area

D) Decreases with increase in temperature and increases with increase in area

Answer: Decreases with increase in temperature and increases with increase in area

Solution: The viscous force is the fluid analog of the sliding friction force between two solid surfaces. For this reason, viscosity is often referred to as fluid friction. Like other frictional forces, viscous forces oppose the relative motion of adjacent fluid layers.

$$F_v = \eta A \frac{dv}{dx}$$

Viscous force is increases with increase in area and decreases with increase in temperature.

Q.32. Interhalogen compound formed by the reaction of bromine with excess of F_2 followed by hydrolysis give bromine containing compound ______.

A) Hypohalate

B) Bromate

C) Perbromate

D) Bromite

Answer: Bromate

Solution: The interhalogen compounds can be prepared by the direct combination or by the action of halogen on lower interhalogen compounds. The product formed depends upon some specific conditions, For e.g.,

$$\underset{\text{(diluted with water)}}{Br_2 + 3F_2} \rightarrow 2BrF_3$$

$$Br_2 + \underset{\text{(excess)}}{5F_2} \rightarrow 2BrF_5$$

$$BrF_5 + 3H_2O \rightarrow HBrO_3 + 5HF$$

$HBrO_3$ is bromic acid, hence, the salt is bromate.

Q.33. The ionic radii of K^+, Na^+, Al^{3+} and Mg^{2+} are in the order :

A) $Na^+ < K^+ < Mg^{2+} < Al^{3+}$

B) $Al^{3+} < Mg^{2+} < K^+ < Na^+$

C) $Al^{3+} < Mg^{2+} < Na^+ < K^+$

D) $K^+ < Al^{3+} < Mg^{2+} < Na^+$

Answer: $Al^{3+} < Mg^{2+} < Na^+ < K^+$

Solution: Al^{3+}, Mg^{2+} and Na^+ are isoelectronic ionic species. For monatomic ionic isoelectronic species as positive charge increases ionic size decreases.

The order of size of Na^+ & K^+ is $Na^+ < K^+$,

$\therefore$ Increasing order of ionic radii: $Al^{3+} < Mg^{2+} < Na^+ < K^+$

Q.34. Which of the following is correct option for A and B according to following reactions?

$$BF_3 + NaH \rightarrow A + NaF$$

$$A + (Me)_3N \rightarrow B$$

A) $A = BH_3;\ B = Me_3N.BF_3$

B) $A = B_2H_6;\ B = Me_3N.BH_3$

C) $A = B_2H_6;\ B = B(CH_3)_3$

D) $A = BH_3;\ B = (BN)_x$

Answer: $A = B_2H_6;\ B = Me_3N.BH_3$

Solution: $$2BF_3 + 6NaH \rightarrow \underset{(A)}{B_2H_6} + 6NaF$$

In this reaction hydride ions replace the fluoride ions. and diborane is formed, which is a Lewis acid due to incomplete octet. It breaks symmetrically with bases like tertiary amines.

$$B_2H_6 + 2Me_3N \rightarrow \underset{(B)}{2(Me_3N)^+} \quad (BH_3)^-$$

Q.35. How many of the following ions will release H_2 gas on reacting with mineral acid?

Ti^{2+}, V^{2+}, Cr^{2+}, Co^{3+}, Mn^{2+}, Zn^{2+}

A) 2

B) 3

C) 4

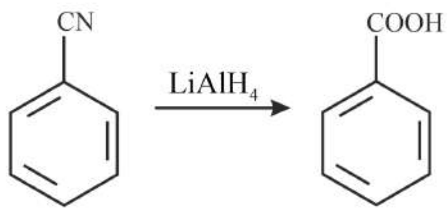
D) 5

Answer: 3

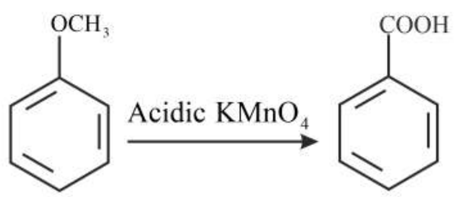
Solution: The ions with stable oxidation states cannot liberate hydrogen gas. Hence, only dipositive cations of titanium, vanadium and chromium release hydrogen gas on reaction with mineral acid.

Q.36. Which of the following reactions represent the correct product?

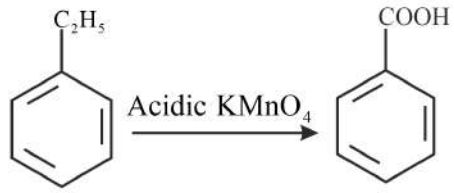
A)



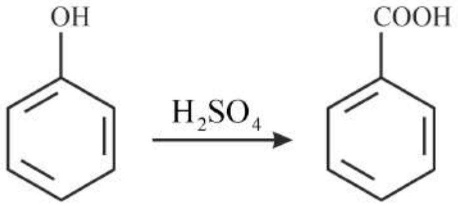
B)



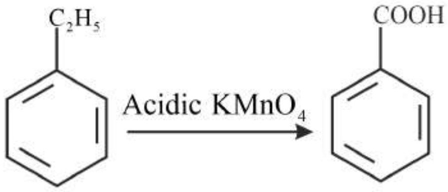
C)



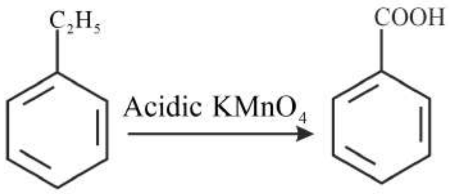
D)



Answer:



Solution: Cyanide on reduction with lithium aluminium hydride gives amine. Anisole doesn't undergo oxidation with potassium permanganate. Alkyl benzenes on oxidation with potassium permanganate give benzoic acid. Phenol undergo sulphonation with sulphuric acid.



Section C: Mathematics

Q.37. If sum and product of mean and variance of a binomial distribution is 24 and 128 respectively, then the probability of no success is

A) $\left(\frac{1}{4}\right)^{16}$

B) $\left(\frac{2}{3}\right)^{16}$

C) $\left(\frac{1}{3}\right)^{16}$

D) $\left(\frac{2}{4}\right)^{16}$

Answer: $\left(\frac{1}{4}\right)^{16}$

Solution: Let number of trails be n and probability of success be p.

Let $q=1-p$

We know that mean and variance will be the roots of the equation

$x^2-24x+128=0$

So, mean $=n\,p=16$ and

variance $=n\,p\,q=8$

Clearly, $p=q=\frac{1}{2}$ and $n=32$

Probability of no success$={}^{32}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{32}=\left(\frac{1}{4}\right)^{16}$

Q.38. Let α,β,γ and δ be the roots of $x^4+x^3+x^2+x+1=0$ then value of $\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}$ is:

A) 2021

B) 1

C) -2021

D) -1

Answer: -1

Solution: We know that $x^4+x^3+x^2+x+1=0$ can be written as

$\frac{x^5-1}{x-1}=0\Rightarrow \quad x^5=1$

i.e. $x=(1)^{\frac{1}{5}}$

So $\alpha,\ \beta,\ \gamma$ and δ are the 5^{th} roots of unity except 1.

Now, from the concept of roots of unity, we know

$$\alpha^n+\beta^n+\gamma^n+\delta^n+1^n=\begin{cases}0 & \text{if } n \text{ is not a multiple of } 5\\ 5 & \text{if } n \text{ is a multiple of } 5\end{cases}$$

So, $\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}=-1$.

Q.39. All words formed by arranging the letters of word 'MANKIND' are placed in dictionary. The rank of word 'MANKIND' is ______ .

A) 1492

B) 1491

C) 1490

D) 1488

Answer: 1492

Solution: Arranging the letters alphabetically, we get

ADIKMNN

When the word starts with any of the letters A/D/I/K, the number of possibilities$=\frac{6!}{2!}\times 4=1440$

Now when the word starts with MA, then

the number of possibilities$=\frac{4!}{2!}\times 3=36$

Now when the word starts with MAN, then the number of possibilities$=3!\times 2=12$

Now when the word starts with MANK, then the number of possibilities$=4$

Hence, rank of the work MANKIND is 1492

Q.40. Let Q be the foot of the tower PQ and R cuts the tower such that $RQ=15\text{ m}$. Let S be any point on ground such that angle of elevation of R from S is 60° and PR makes an angle 15° on S. Then, height of tower is

A) $10\left(\sqrt{3}+1\right)\text{ m}$

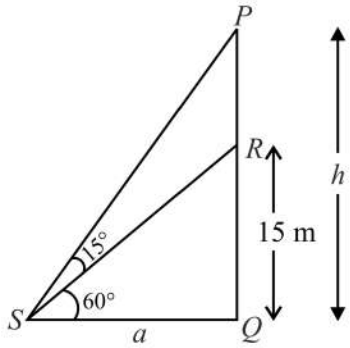
B) $10\left(\sqrt{3}-1\right)\text{ m}$

C) $5\sqrt{3}\left(2+\sqrt{3}\right)\text{ m}$

D) $5\sqrt{3}\left(2-\sqrt{2}\right)\text{ m}$

Answer: $5\sqrt{3}\left(2+\sqrt{3}\right)\text{ m}$

Solution:



From the right triangle RQS,

$\tan 60^{\circ}=\frac{15}{a}$

$a=\frac{15}{\sqrt{3}}=5\sqrt{3}$

In right triangle PQS,

$\tan 75^{\circ}=\frac{h}{a}$

$\Rightarrow \frac{h}{5\sqrt{3}}=2+\sqrt{3}$

$\Rightarrow h=10\sqrt{3}+15$

Q.41. The number of solution of x in $[-4\pi, 4\pi]$ of the equation $|\cos x|=\sin x$ is:

A) 2

B) 4

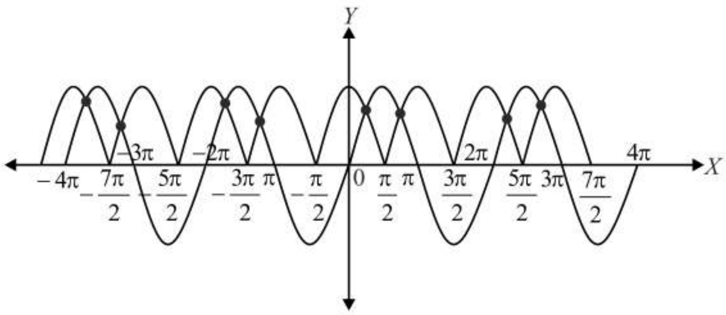
C) 6

D) 8

Answer: 8

Solution: The number of solutions will be the number of points of intersection of the graphs $y=|\cos x|$ & $y=\sin x$

Plotting the two curves on the same axis, we get,



the number of intersection as 8

Q.42. Given curve $C_1 \equiv x^2+(y-1)^2=1$, and locus of centre of circle which touches C_1 and x-axis is curve $C_2(x,y); y>0$, then area bounded by C_2 and line $y=4$ is:

A) $\frac{64}{3}$

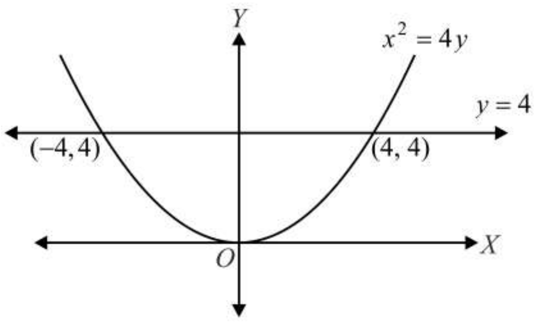
B) $\frac{32}{3}$

C) $\frac{16}{3}$

D) $\frac{50}{3}$

Answer: $\frac{64}{3}$

Solution:



Let the equation of circle which touches C_1 and $x-$axis be: $(x-\alpha)^2+(y-\beta)^2=\beta^2$

where (α,β) is its centre.

As it touches given circle $x^2+(y-1)^2=1$, we have

distance between their centres$=r_1+r_2$

$\Rightarrow \sqrt{\alpha^2+(\beta-1)}^2=|\beta+1|$

$\Rightarrow \alpha^2=4\beta$

$\therefore C_2(x,y)\equiv x^2=4y$

Required Area $=2\int_0^4\left(4-\frac{x^2}{4}\right)dx=2\left[4x-\frac{x^3}{12}\right]_0^4=\frac{64}{3}$

Q.43. Which of the following is tautology?

A) $(\sim p\vee q)\Rightarrow q$

B) $q\Rightarrow(\sim p\vee q)$

C) $(\sim p\vee q)\Rightarrow p$

D) $p \Rightarrow (\sim p \vee q)$

Answer: $q \Rightarrow (\sim p \vee q)$

Solution: By solving all option one by one we get,

(i) $(\sim p \vee q) \Rightarrow q \equiv \sim(\sim p \vee q) \vee q \equiv (p \wedge \sim q) \vee q \equiv p \vee q$

(Using property $A \Rightarrow B = \sim A \vee B$ and $\sim(A \vee B) = \sim A \wedge \sim B$)

(ii) $q \Rightarrow (\sim p \vee q) \equiv \sim q \vee (\sim p \vee q) \equiv (\sim q \vee q) \vee \sim p$

which is a tautology, as $A \Rightarrow B = \sim A \vee B$ and $\sim A \vee A \equiv \text{Tautology}$.

(iii) $(\sim p \vee q) \Rightarrow p \equiv (p \wedge \sim q) \vee p \Rightarrow p \wedge (p \vee \sim q)$

(Using property $A \Rightarrow B = \sim A \vee B$)

(iv) $p \Rightarrow (\sim p \vee q) \equiv \sim p \vee (\sim p \vee q) \Rightarrow \sim p \vee q$

(Using property $A \Rightarrow B = \sim A \vee B$)

Q.44. If a fair die is thrown two times and α, β are numbers appearing on it, then probability of $x^2 + \alpha x + \beta > 0$ is:

A) $\frac{1}{2}$

B) $\frac{14}{36}$

C) $\frac{4}{9}$

D) $\frac{17}{36}$

Answer: $\frac{17}{36}$

Solution: Total sample space of two dice will be equal to $6 \times 6 = 36$

Now for $x^2 + \alpha x + \beta > 0$ its discriminant D must be less than zero,

So $D = b^2 - 4ac < 0$ or $\alpha^2 - 4\beta < 0 \Rightarrow \alpha^2 < 4\beta$

Now, if $\alpha = 1, \beta$ can take values $1, 2, 3, 4, 5, 6$

If $\alpha = 2, \beta$ can take values $2, 3, 4, 5, 6$

If $\alpha = 3, \beta$ can take values $3, 4, 5, 6$

If $\alpha = 4, \beta$ can take values $5, 6$

If $\alpha = 5$ and 6, no value of β possible

So number of favourable ways $= 17$

Required probability$= \frac{\text{favourable ways}}{\text{total sample space}} = \frac{17}{36}$

Q.45. Let $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4, 5, 6\}$, then number of functions for which $f(1) + f(2) = f(3)$ is:

A) 108

B) 90

C) 15

D) 18

Answer: 90

Solution: Given, $f(1)+f(2)=f(3)$

Now possible values of $f(3)$ are $2,3,4,5$ and 6 (it cannot be 1 as minimum value $f(1)$ & $f(2)$ can take is1, so $1+1=2$)

For each k, if $f(3)=k$ then there will be $(k-1)$ set of values for $f(1)$ and $f(2)$

For example if $f(3)=3$ then possible value of $f(1)$ & $f(2) \Rightarrow (1,2)$ or $(2,1) \Rightarrow 2$ values

Similarly for $f(3)=4$, possible values will be $(1,3),(3,1),(2,2) \Rightarrow 3$ values,

And so on, so number of possible combinations of $f(1),\ f(2)$ and $f(3)$ are $1+2+3+4+5=15$

And for each value of $f(3)$ there will be 6 possible value of $f(4)$, so total cases will be $15\times 6=90$

Q.46. If $\lim_{x\to\infty}\sqrt{x^2-x-1}+\alpha x+\beta=0$, then $|8(\alpha+\beta)|$ is

A) 8

B) 16

C) 4

D) 24

Answer: 4

Solution: Given, $\lim_{x\to\infty}\sqrt{x^2-x-1}+\alpha x+\beta=0$

Now rewriting the expression we get,

$$\lim_{x\to\infty}\left(x^2-x-1\right)^{\frac{1}{2}}+\alpha x=-\beta$$

$$\Rightarrow \lim_{x\to\infty} x\left(1-\frac{1}{x}-\frac{1}{x^2}\right)^{\frac{1}{2}}+\alpha x=-\beta$$

Now on using binomial approximation $(1+x)^n=1+nx$ we get,

$$\lim_{x\to\infty} x\left(1-\frac{1}{2x}-\frac{1}{2x^2}\right)+\alpha x=-\beta$$

$$\Rightarrow \lim_{x\to\infty} x-\frac{1}{2}-\frac{1}{2x}+\alpha x=-\beta$$

$$\Rightarrow \lim_{x\to\infty} x(1+\alpha)-\frac{1}{2}-\frac{1}{2x}=-\beta$$

Now for limit to exist $(1+\alpha)=0 \Rightarrow \alpha=-1$

Now on putting the value of α we get,

$$\lim_{x\to\infty} x(1+\alpha)-\frac{1}{2}-\frac{1}{2x}=-\beta$$

$$\Rightarrow \lim_{x\to\infty} -\frac{1}{2}-\frac{1}{2x}=-\beta$$

$$\Rightarrow -\frac{1}{2}-0=-\beta$$

$$\Rightarrow \beta=\frac{1}{2}$$

So, $|8(\alpha+\beta)|=\left|8\left(-1+\frac{1}{2}\right)\right|=8\times\frac{1}{2}=4$

Q.47. Argument of $z=1+e^{i\frac{6\pi}{5}}$ is equal to:

A) $\frac{2\pi}{5}$

B) $\frac{3\pi}{5}$

C) $\frac{7\pi}{5}$

D) $-\frac{2\pi}{5}$

Answer: $-\frac{2\pi}{5}$

Solution: Given,

$z=1+e^{i\frac{6\pi}{5}}$

Now using the formula $e^{i\theta}=\cos\theta+i\sin\theta$ we get,

$z=1+\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}$

Now by using the formula $\left(1+\cos 2\theta=2\cos^2\theta\right)$ and $\sin 2\theta=2\sin\theta\cos\theta$, we get

$z=2\cos^2\frac{3\pi}{5}+i2\sin\frac{3\pi}{5}\cos\frac{3\pi}{5}$

Now, $z=2\cos\frac{3\pi}{5}\left(\cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}\right)$

(As $\frac{3\pi}{5}$ is in 2^{nd} quadrant so we will use $\cos(\pi-\theta)=-\cos\theta$ & $\sin(\pi-\theta)=\sin\theta$ formula to simplify)

So, we get $z=2\cos\frac{2\pi}{5}\left(\cos\left(-\frac{2\pi}{5}\right)+i\sin\left(-\frac{2\pi}{5}\right)\right)$

So, $\arg(z)=-\frac{2\pi}{5}$

Q.48. Given $a_1=b_1=1, a_n=a_{n-1}+2$ and $b_n=a_n+b_{n-1}$. Then, the value of $\sum_{n=1}^{15}a_n\cdot b_n$ is ______ .

A) 2750

B) 27560

C) 27550

D) None of these

Answer: 27560

Solution: Given $a_n-a_{n-1}=2$

So the sequence is an A.P. with first term $a_1=1$ and common difference $d=2$

i.e., $a_n=a_1+(n-1)d=2n-1$

Also, $a_n=b_n-b_{n-1}$

We know $2n-1=n^2-(n-1)^2$

So, $b_n=n^2$

Now, $\sum_{n=1}^{15}a_n\cdot b_n=\sum_{n=1}^{15}\left(2n^3-n^2\right)=2\sum_{n=1}^{15}n^3-\sum_{n=1}^{15}n^2$

$=2\left(\frac{15\cdot16}{2}\right)^2-\frac{15\cdot16\cdot31}{6}$

$=27560$

Q.49. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

A) $x+2y-2z=0$

B) $3x+2y-2z=0$

C) $x-2y+z=0$

D) $5x+2y-4z=0$

Answer: $x-2y+z=0$

Solution: Plane $1: ax+by+cz=0$ contains line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$

$\therefore 2a+3b+4c=0 \quad \ldots\text{(i)}$

Plane $2: a'x+b'y+c'z=0$ is perpendicular to plane containing lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$

$\therefore 3a'+4b'+2c'=0\ \&\ 4a'+2b'+3c'=0$

$\Rightarrow \frac{a'}{12-4}=\frac{b'}{8-9}=\frac{c'}{6-16}$

$\Rightarrow 8a-b-10c=0 \quad \ldots\text{(ii)}$

From (i) and (ii),

$\frac{a}{-30+4}=\frac{b}{32+20}=\frac{c}{-2-24}$

$\Rightarrow$ Equation of plane $x-2y+z=0$.

Q.50. Let a, b be two non-zero real numbers. If $p,\ r$ are roots of $x^2-8ax+2a=0$ and q, s are the roots of $x^2+12bx+6b=0$ such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then the value of $a^{-1}-b^{-1}$ is equal to _____ .

A) -2

B) 10

C) 38

D) -58

Answer: 38

Solution: Since $p,\ r$ are roots of $x^2-8ax+2a=0$ and

q, s are the roots of $x^2+12bx+6b=0$

So the equation $2ax^2-8ax+1=0$ has roots $\frac{1}{p}, \frac{1}{r}$ and

$6bx^2+12bx+1=0$ has roots $\frac{1}{q}, \frac{1}{s}$.

Given that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P.

Let $\frac{1}{p}=\alpha-3\beta, \frac{1}{q}=\alpha-\beta, \frac{1}{r}=\alpha+\beta$ and $\frac{1}{s}=\alpha+3\beta$

i.e. $\frac{1}{p}+\frac{1}{r}=2\alpha-2\beta=4$ and

$\frac{1}{q}+\frac{1}{s}=2\alpha+2\beta=-2$

or $\alpha=\frac{1}{2},\ \beta=-\frac{3}{2}$

i.e. $\frac{1}{p}=5,\ \frac{1}{q}=2,\ \frac{1}{r}=-1,\ \frac{1}{s}=-4$

Now

$\frac{1}{p}\times\frac{1}{r}=\frac{1}{2a}=-5 \Rightarrow \frac{1}{a}=-10$ and $\frac{1}{q}\times\frac{1}{s}=\frac{1}{6b}=-8 \Rightarrow \frac{1}{b}=-48$

Hence, $a^{-1}-b^{-1}=-10+48=38$

Q.51. The remainder of $(2024)^{2024}$ when divided by 7 is

A) 1

B) 2

C) 5

D) 3

Answer: 1

Solution:

Given, $(2024)^{2024}$

Now on rearranging we get,

$(2024)^{2024}=(7\times 289+1)^{2024}$

Now on using binomial expansion we get,

$(7\times 289+1)^{2024}={}^{2024}C_0(7\times 289)^{2024}+{}^{2024}C_1(7\times 289)^{2023}\times 1+\cdots$

$+{}^{2024}C_{2023}(7\times 289)\times 1^{2023}+{}^{2024}C_{2024}(1)^{2024}$

$=7k+1$

So we can clearly see that $(7\times 289+1)^{2024}$ is in the form of $7k+1$, so remainder on divide by 7 will be 1

$\therefore$ Remainder $=1$