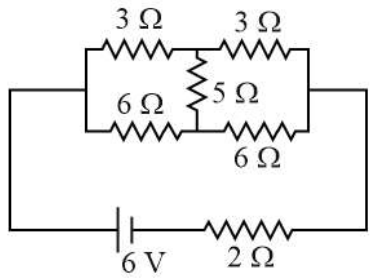
JEE Main Exam 2022 - Session 2

26 Jul 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. For the given circuit diagram, the current supplied by the battery will be___



A) $1\ \text{A}$

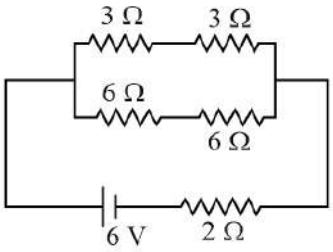
B) $2\ \text{A}$

C) $0\ \text{A}$

D) $3\ \text{A}$

Answer: $1\ \text{A}$

Solution: The given circuit in the question is a case of a balanced Wheatstone bridge. Therefore, the circuit can be simplified as given below.



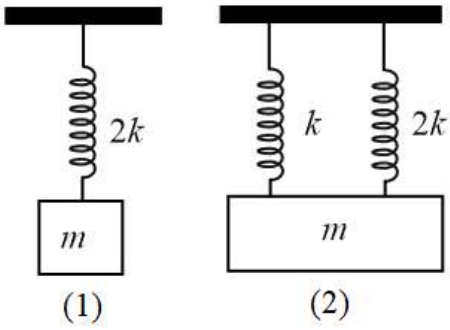
The equivalent resistance for the given circuit will be,

$$R_{\text{eq}} = \frac{(3+3)\times(6+6)}{(3+3)+(6+6)} + 2 = 6\ \Omega$$

Therefore, the current supplied by the battery is

$$i = \frac{V}{Req} = \frac{6}{6} = 1\ \text{A}$$

Q.2. The ratio of time period of oscillation of system 1 to that of system 2 is,



A) $\sqrt{3}$

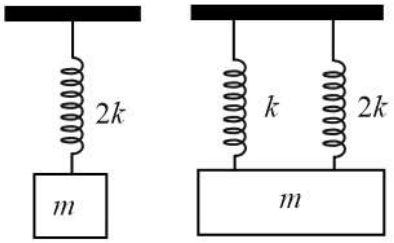
B) $\sqrt{\frac{2}{3}}$

C) $\sqrt{\frac{3}{2}}$

D) $\sqrt{\frac{1}{3}}$

Answer: $\sqrt{\frac{3}{2}}$

Solution:



Time period of a spring block system is given by,

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{k}}$ Here, k is the force constant of the system

For system 1 :

$k_{eq} = 2k$

For system 2 :

The springs are connected in parallel. Therefore,

$k_{eq} = 2k + k = 3k$

$\frac{T_1}{T_2} = \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{3k}{2k}} = \sqrt{\frac{3}{2}}$

Q.3. Particle A and particle B are projected at angle 45° and 30° respectively, with the same projection speed. Ratio of the range of two particles i.e. $\frac{R_A}{R_B}$ is

A) $\frac{1}{2}$

B) $\sqrt{\frac{3}{2}}$

C) $\frac{2}{\sqrt{3}}$

D) $\frac{2}{1}$

Answer: $\frac{2}{\sqrt{3}}$

Solution: The range of a projectile projected with initial velocity u and angle of projection θ is given by,

$$R = \frac{u^2 \sin 2\theta}{g}$$

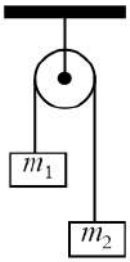
$$= \frac{u^2 \sin(2\theta_A) \times g}{g \times u^2 \sin(2\theta_B)}$$

$$= \frac{\sin(2 \times 45^\circ)}{\sin(2 \times 30^\circ)}$$

$$= \frac{\sin(90^\circ)}{\sin(60^\circ)}$$

$$\Rightarrow \frac{R_A}{R_B} = \frac{2}{\sqrt{3}}$$

Q.4. In the shown system in case(1) $m_1 = 2m_2$, while in case(2) $m_1 = 3m_2$, then acceleration in case(1) would be how many times that of acceleration in case(2) when set free.



A) $\frac{1}{3}$

B) $\frac{3}{2}$

C) 3

D) $\frac{2}{3}$

Answer: $\frac{2}{3}$

Solution: In both cases $m_1 > m_2$. Therefore, m_1 will accelerate downward and m_2 will accelerate upward.

For m_1,

$m_1 g - T = m_1 a$

For m_2,

$T - m_2 g = m_2 a$

Adding the above equations we get,

$a = \frac{m_1 - m_2}{m_1 + m_2} g$

$\Rightarrow a_1 = \frac{m_1 - m_2}{m_1 + m_2} g$
$= \frac{2m_2 - m_2}{2m_2 + m_2} g$
$= \frac{g}{3}$

and

$\Rightarrow a_2 = \frac{m_1 - m_2}{m_1 + m_2} g$
$= \frac{3m_2 - m_2}{3m_2 + m_2} g$
$= \frac{g}{2}$

$\Rightarrow a_1 = \frac{2}{3} a_2$

Q.5. The velocity of particle is $\left(\frac{1}{3}\right)^{rd}$ of the escape velocity. Find the maximum height reached by the body. $(R = 6400 \text{ km})$

A) 6400 km

B) 1200 km

C) 1600 km

D) 800 km

Answer: 800 km

Solution: Total mechanical energy of the body at the surface of earth

$M_i = -\frac{GMm}{R} + \frac{1}{2} m \left[\frac{1}{3}\left(\sqrt{\frac{2GM}{R}}\right)\right]^2$
$= -\frac{GMm}{R} + \frac{GMm}{9R}$
$= -\frac{8GMm}{9R}$

Total mechanical energy of the body at the highest point

$M_f = -\frac{GMm}{R+h}$

Using conservation of mechanical energy,

$M_i = M_f$
$\Rightarrow -\frac{GMm}{R+h} = -\frac{8GMm}{9\text{R}}$

$\Rightarrow R + h = \frac{9R}{8}$

$\Rightarrow h = \frac{R}{8} = \frac{6400}{8} = 800 \text{ km}$

Q.6. Two charged spherical conductors are charged and then connected by a conducting wire. At the equilibrium, the ratio of the electric field on the surface of first sphere to that of the second sphere is (radius of first sphere $= 5$ cm, radius of second sphere $= 10$ cm)

A) $1:2$

B) $1:4$

C) $4:1$

D) $2:1$

Answer: $2:1$

Solution: When two conductors are connected by a conducting wire, their potentials become equal. Therefore, $V_1 = V_2$

The potential of a spherical conductor is given by, $V = \frac{kQ}{R}$

$\Rightarrow \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$

$\Rightarrow Q \propto R$

Now the electric field near the surface of the conductor will be, $E = \frac{kQ}{R^2}$

$\Rightarrow \frac{E_1}{E_2} = \frac{\frac{kQ_1}{R_1^2}}{\frac{kQ_2}{R_2^2}} = \frac{R_2}{R_1}$

$\Rightarrow \frac{E_1}{E_2} = \frac{10}{5} = \frac{2}{1}$

Q.7. If the ratio of mass number of two nuclei is $\frac{4}{3}$, then ratio of their radii would be,

A) $\frac{4}{3}$

B) $\frac{3}{4}$

C) $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

D) $\left(\frac{3}{4}\right)^{\frac{2}{3}}$

Answer: $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

Solution: Given: $\frac{A_1}{A_2} = \frac{4}{3}$

The radius of a nucleus is proportional to the cube root of the mass number, i.e. $R \propto (A)^{\frac{1}{3}}$.

$\Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{4}{3}\right)^{\frac{1}{3}}$

Q.8. A particle at rest breaks down in two parts of mass $\frac{M}{3}$ and $\frac{2M}{3}$. The ratio of de-Broglie wavelength of two parts is equal to

A) $2:1$

B) $1:1$

C) $1:3$

D) $1:2$

Answer: $1:1$

Solution: Magnitude of the momentum of both the particle is same to maintain the net momentum of system equal to zero. So ratio of momentum as well as de- Broglie wavelength would be equal to 1.

Q.9. In an AM signal, maximum amplitude of wave is $60\ \mathrm{V}$, while minimum amplitude is $20\ \mathrm{V}$. The percentage modulation index is equal to

A) 24%

B) 50%

C) 30%

D) 33%

Answer: 50%

Solution: If amplitude of carrier wave is A_c and amplitude of message wave is A_m,

$A_{max} = A_c + A_m$ and $A_{min} = A_c - A_m$

Modulation index,

$$M_i = \frac{Am}{Ac} = \frac{\frac{Amax - A_{min}}{2}}{\frac{Amax + A_{min}}{2}} = \frac{60-20}{60+20} = \frac{1}{2} = 50\%$$

Q.10. The ball of mass $1.5\ \mathrm{kg}$ hits the wall with a speed of $24\ \mathrm{m\ s^{-1}}$ and without change in magnitude it reverses back. Force exerted was $100\ \mathrm{N}$, then the time of impact of ball with wall was $\ldots\ldots \times 10^{-2}\ \mathrm{s}$.

A) $72 \times 10^{-2}\ \mathrm{s}$

B) $54 \times 10^{-2}\mathrm{s}$

C) $36 \times 10^{-2}\mathrm{s}$

D) $18 \times 10^{-2}\mathrm{s}$

Answer: $72 \times 10^{-2}\ \mathrm{s}$

Solution: Given: $m = 1.5\ \mathrm{kg}\ \&\ u = 24\ \mathrm{m\ s^{-1}}$

Change in momentum of the ball$= 2mv$

$= 2 \times 1.5 \times 24$

$= 72\ \mathrm{N\ s}$

As the force applied during collision is equal to $100\ \mathrm{N}$ and if t is the duration of collision, so

$100 \times t = \Delta p$

$\Rightarrow t = \frac{72}{100}\ \mathrm{s}$

$t = 72 \times 10^{-2}\ \mathrm{s}$

Q.11. A cube of ice of dimensions $(60\ \mathrm{cm} \times 50\ \mathrm{cm} \times 20\ \mathrm{cm})$ is enclosed by a wall of thickness $1\ \mathrm{cm}$ and conductivity $0.05\ \mathrm{W\ cm^{-1}\ ^\circ C^{-1}}$ with surrounding temperature $40^\circ\mathrm{C}$, the rate of melting ice is equal to, $\left(L_{\text{fusion}} = 80\ \mathrm{cal\ g^{-1}}\right)$

A) $30\ \mathrm{g\ s^{-1}}$

B) $62\ \mathrm{g\ s^{-1}}$

C) $80\ \mathrm{g\ s^{-1}}$

D) $94\ \mathrm{g\ s^{-1}}$

Answer: $62\ \mathrm{g\ s^{-1}}$

Solution: Total area $= 2\left(60 \times 50 + 50 \times 20 + 20 \times 60\right)\ \text{cm}^2$

$= 10400\ \text{cm}^2$

As we know,

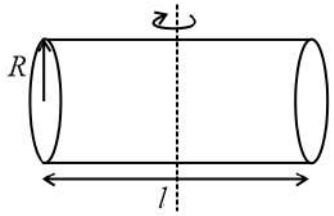
$\frac{dQ}{dt} = \frac{KA\Delta\theta}{l}$

$\Rightarrow \frac{dm}{dt}L = \frac{KA\Delta\theta}{d}$

$\Rightarrow \frac{dm}{dt} \times 80 = \frac{0.05}{4.2} \times \frac{10400 \times 40}{1}$

$\Rightarrow \frac{dm}{dt} = 62\ \text{g s}^{-1}$

Q.12. The moment of inertia of solid cylinder (mass m) about the shown axis is equal to



A) $\frac{m}{3}\left(R^2 + l^2\right)$

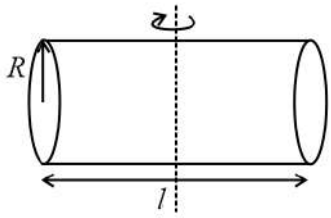
B) $\frac{m}{4}\left(R^2 + \frac{l^2}{3}\right)$

C) $\frac{m}{4}\left(R^2 + l^2\right)$

D) $\frac{m}{3}\left(R^2 + \frac{l^2}{4}\right)$

Answer: $\frac{m}{4}\left(R^2 + \frac{l^2}{3}\right)$

Solution:



Moment of Inertia $= \frac{ml^2}{12} + \frac{mR^2}{4}$

$= \frac{m}{4}\left(\frac{l^2}{3} + R^2\right)$

Q.13. An ion is given as ${}^{48}_{22}X^{3+}$, then the difference between the number of neutrons and number of electrons is

A) 11

B) 4

C) 7

D) 2

Answer: 7

Solution: Number of neutrons $= 48 - 22 = 26$

Number of electrons $= 22 - 3 = 19$

$\Rightarrow \quad x = 26 - 19 = 7$

Q.14. Find out elongation in load due to self weight in terms of $(M, Y, L \text{ and } A)$.

A) $\frac{MgL}{AY}$

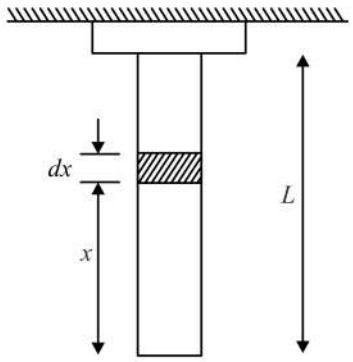
B) $\frac{MgL}{2AY}$

C) $\frac{3MgL}{2AY}$

D) $\frac{2MgL}{3AY}$

Answer: $\frac{MgL}{2AY}$

Solution:



Elongation in the load due to self weight,

$= \int_0^L \left(\frac{Mgx}{L}\right) \times \frac{1}{AY} dx$

$= \frac{MgL^2}{2AYL}$

$= \frac{MgL}{2AY}$

Q.15. Ball A has its mass $\frac{2}{3}$ times that of Ball B. If same force is applied to both the balls then ratio of acceleration of ball A to that of ball B is equal to

A) $\frac{1}{3}$

B) $\frac{2}{3}$

C) $\frac{3}{2}$

D) $\frac{1}{2}$

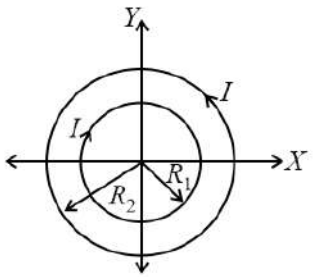
Answer: $\frac{3}{2}$

Solution: As we know, $F = ma$.

Therefore,

$\frac{a_A}{a_B} = \frac{m_B}{m_A} = \frac{3}{2}$

Q.16. Two concentric coils with radii $R_1 = 30\ \text{cm}$ and $R_2 = 50\ \text{cm}$ are placed in XY plane with $I = 7\ \text{A}$. Net magnetic moment is equal to



A) $1.16\ \text{A m}^2\ \hat{\text{k}}$

B) $3.52\ \text{A m}^2\ \hat{\text{k}}$

C) $-1.16\ \text{A m}^2\ \hat{\text{k}}$

D) $-3.52\ \text{A m}^2\ \hat{\text{k}}$

Answer: $3.52\ \text{A m}^2\ \hat{\text{k}}$

Solution: $\overrightarrow{M} = I\left(\pi R_2^2 - \pi R_1^2\right)\hat{\text{k}}$

$\overrightarrow{M} = 7 \times \frac{22}{7} \times (40)^2 \times 10^{-4}\,\hat{\text{k}}$

$= \frac{22 \times 16}{100}\,\hat{\text{k}}$

$= 3.52\ \text{A m}^2\ \hat{\text{k}}$

Section B: Chemistry

Q.17. Which group 13 element has lowest melting point and is close to a metalloid?

A) Al

B) Ga

C) In

D) Tl

Answer: Ga

Solution: The expected order of decrease in melting points in $\mathrm{B} > \mathrm{Al} > \mathrm{Ga} > \mathrm{In} > \mathrm{Tl}$. But this order is incorrect. Due to structure changes, melting point instead of decreasing, increases from Ga to Tl and as such Ga has the lowest melting point. Hence the actual order is $\mathrm{B} > \mathrm{Al} > \mathrm{Tl} > \mathrm{In} > \mathrm{Ga}$

Q.18. Consider the following statements:

Statement-1: Boric acid is a weak acid in aqueous solution.

Statement-2 : It acts as a Lewis acid due to the presence of incomplete octet of boron

A) Both statements are correct

B) Statement-1 is correct and statement-2 is incorrect

C) Statement-1 is incorrect and statement-2 is correct

D) Both the statements are incorrect

Answer: Both statements are correct

Solution: Boric acid is a weak acid because it does not completely ionize in water or other aqueous solution. It is not able to release H^{+} ions on its own because firstly it receives hydroxide ions $\left(\mathrm{OH}^{-}\right)$ from water molecule in order to complete its octet and then it releases H^{+} ions.

Q.19. Which of the following compound shows highest spin only magnetic moment?

A) MnF_2

B) MnF_3

C) MnF_4

D) MnO_2

Answer: MnF_2

Solution: Spin only magnetic moment can be calculated as

$\mu = \sqrt{\mathrm{n}(\mathrm{n}+2)}\,\mathrm{BM}$

$\mathrm{n} = \text{number of unpaired electrons}$

$\mathrm{Mn}^{+2}\left(3\mathrm{d}^5\right) = \sqrt{5(5+2)} = \sqrt{35}\ \mathrm{B.M}\ \text{in}\ \mathrm{MnF}_2$

$\mathrm{Mn}^{+3}\left(3\mathrm{d}^4\right) = \sqrt{24}\ \mathrm{B.M}\ \text{in}\ \mathrm{MnF}_3$

$\mathrm{Mn}^{+4}\left(3\mathrm{d}^3\right) = \sqrt{15}\ \mathrm{B.M}\ \text{in}\ \mathrm{MnF}_4$

$\mathrm{Mn}^{+4}\left(3\mathrm{d}^3\right) = \sqrt{15}\ \mathrm{B.M}\ \text{in}\ \mathrm{MnO}_2$

Q.20. Which among the following is a broad spectrum antibiotic drug?

A) Ofloxacin

B) Penicillin G

C) Novestrol

D) Terpineol

Answer: Ofloxacin

Solution: The range of bacteria or other microorganisms that are affected by a certain antibiotic is expressed as its spectrum of action. Antibiotics which kill or inhibit a wide range of Gram-positive and Gram-negative bacteria are said to be broad spectrum antibiotics. Those effective mainly against Gram-positive or Gram-negative bacteria are narrow spectrum antibiotics. Penicillin G has a narrow spectrum.

Vancomycin and ofloxacin are the important broad spectrum antibiotics.

Q.21. Which among the following metals is not extracted from sulphide ore?

A) Al

B) Fe

C) Zn

D) Cu

Answer: Al

Solution: The sulphide ore of iron is iron pyrite.

Copper Glance (Cu_2S) is a sulphide ore of copper.

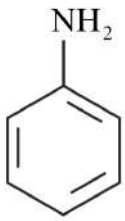
The sulphide containing ore of zinc is zinc blende (ZnS).

Aluminium cannot be obtained by chemical reduction due to its strong affinity for oxygen.

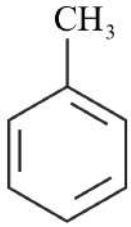
Q.22. Which of the following is not a benzenoid?

A)

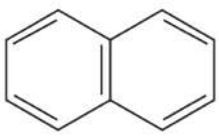
B)



C)

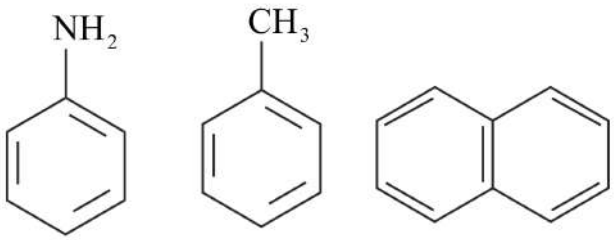


D)



Answer:

Solution: Compounds that contain at least one benzene ring in their structure are called benzenoid compounds.



The above compounds are benzenoid compounds due to the presence of benzene ring.

Q.23. Vulcanised rubber is prepared from:

A) Styrene + isoprene

B) Isoprene + sulphur

C) Neoprene + sulphur

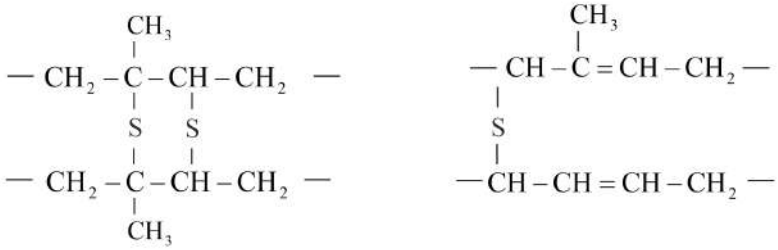
D) Neoprene + styrene

Answer: Isoprene + sulphur

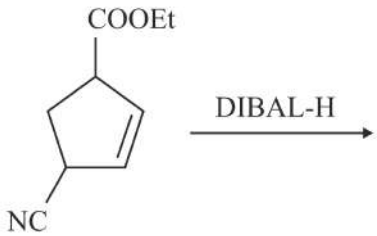
Solution: Natural rubber may be considered as a linear polymer of isoprene (2-methyl-1, 3-butadiene) and is also called as cis - 1, 4 - polyisoprene.

To improve the physical properties, a process of vulcanisation is carried out. This process consists of heating a mixture of raw rubber with sulphur and an appropriate additive at a temperature range between 373 K to 415 K.

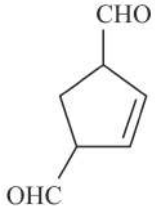
In the manufacture of tyre rubber, 5% of sulphur is used as a cross linking agent. The probable structures of vulcanised rubber molecules are depicted below:



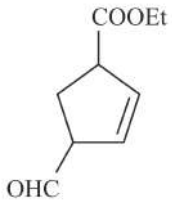
Q.24. Which of the following is the product of the given reaction?



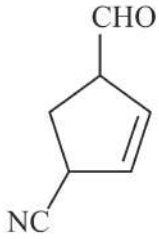
A)



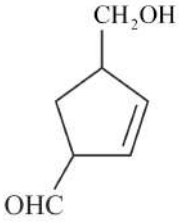
B)



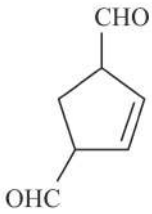
C)



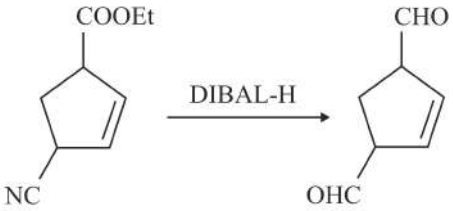
D)



Answer:



Solution:



$DIBAL-H$ can reduce both ester and cyanide group into aldehyde.

Q.25. Haemoglobin contains 0.34% of iron (by mass). What mass of iron (in mg) is present in $33\ gm$ of haemoglobin? (Round off to the nearest integer)

A) $224\ mg$

B) $112\ mg$

C) $56\ mg$

D) $72\ mg$

Answer: $112\ mg$

Solution: Mass of Fe present in $100\ gm$ of $Hb = 0.34\ gm$

Mass of Fe present in $33\ gm$ of $Hb = \frac{0.34 \times 33}{100}$

$= 0.1122\ gm$

$= 112\ mg$

Q.26. $H_2F_2(g) \rightarrow H_2(g) + F_2(g)$

If ΔU for the above reaction is $-59.6\ kJ\ mol^{-1}$ at 27^oC, then find the value of ΔH at the same temperature. $\left(R = 8.314\ J\ K^{-1}\ mol^{-1}\right)$

(Consider magnitude of ΔH only)

A) 57.11

B) 62.09

C) 59.6

D) 67.31

Answer: 57.11

Solution: $\Delta H = \Delta U + \left(\Delta n_g\right)RT$

$\Delta n_g = 2 - 1$

$= -59.6 + 1 \times \frac{8.314}{1000} \times 300$

$= -57.11$

Q.27. Which of the following is known as animal starch?

A) Glycogen

B) Starch

C) Sucrose

D) None of these

Answer: Glycogen

Solution: The carbohydrates are stored in animal body as glycogen. It is also known as animal starch because its structure is similar to amylopectin and is rather more highly branched. It is present in liver, muscles and brain. When the body needs glucose, enzymes break the glycogen down to glucose.

Q.28. Consider the following statements:

Assertion: LiF is sparingly soluble in water.

Reason: Radius of Li^+ is the least among its group members and hence its hydration enthalpy is very less.

A) Assertion is correct, reason is correct and reason is the correct explanation for assertion

B) Assertion is correct, reason is correct and reason is not the correct explanation for assertion

C) Assertion is correct and reason is incorrect

D) Assertion is incorrect and reason is correct

Answer: Assertion is correct and reason is incorrect

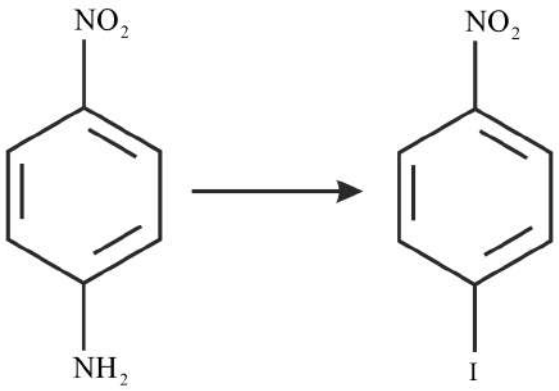
Solution: The low solubility of LiF in water is due to its high lattice enthalpy.

The hydration enthalpies of alkali metal ions decrease with increase in ionic sizes.

$Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

Li^+ has maximum degree of hydration and for this reason lithium salts are mostly hydrated, e.g., $LiCl \cdot 2H_2O$

Q.29. Correct set of reagent for the following conversion is:



A) $NaNO_2/HCl,\ KI$

B) $KI,\ NaNO_2/HCl$

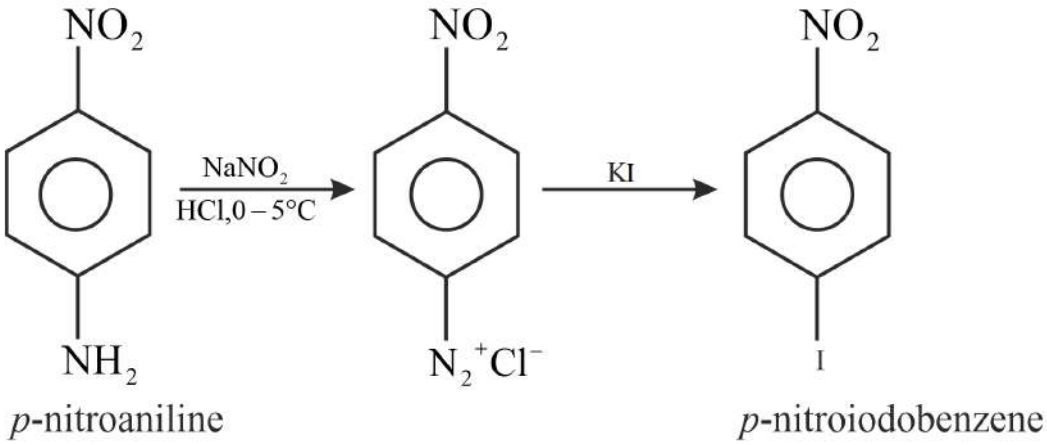
C) $Fe/HCl,\ NaNO_2/HCl,\ KI$

D) $NaNO_2/HCl,\ CH_3I$

Answer: $NaNO_2/HCl,\ KI$

Solution:

The product formation takes by diazotisation of amine group followed by aromatic nucleophilic substitution reaction of iodide.



Q.30. A is a non-volatile solute. If for 1 molal solution, ΔT_b is $3\ K$ and for 2 molal solution, ΔT_f is $6\ K$, then find the ratio of k_b and k_f.

A) $1:2$

B) $1:3$

C) $1:1$

D) $1:4$

Answer: $1:1$

Solution: $3\ K = \Delta T_b = ik_b \times m = i \times k_b \times 1$

$6K = \Delta T_f = i \times k_f \times m = i \times k_f \times 2$

$\frac{k_b}{k_f} = 1$

Q.31. (A) Assertion: Phenolphthalein is an organic indicator.

(R) Reason: Phenolphthalein is a weak acid and does not dissociate in base.

A) A and R both are correct and R is correct explanation of A

B) A and R both are correct and R is not correct explanation of A

C) A is correct but R is not correct

D) A is incorrect but R is correct

Answer: A is correct but R is not correct

Solution: Phenolphthalein, $(C_{20}H_{14}O_4)$, an organic compound of the phthalein family that is widely employed as an acid-base indicator. As an indicator of a solution's pH, phenolphthalein is colourless below pH 8.5 and attains a pink to deep red hue above pH 9.0.

Q.32. Which of the following complex is diamagnetic in nature?

A) $K_3[Fe(CN)_6]$

B) $K_4[Fe(CN)_6]$

C) $K_2[Cu(CN)_4]$

D) None of these

Answer: $K_4[Fe(CN)_6]$

Solution: The species with zero unpaired electrons are diamagnetic.

$[Fe(CN)_6]^{3-}\left(Fe^{+3}=3d^5\right)$-One unpaired electron is left after pairing.

$[Fe(CN)_6]^{4-}\left(Fe^{+2}=[Ar]3d^6\right)$- No unpaired electron is left after pairing.

$[Cu(CN)_4]^{2-}\left(Cu^{2+}=[Ar]3d^9\right)$-one unpaired electron is present.

Q.33. Arrange the following in increasing order of covalent character:

A) $CaI_2<CaBr_2<CaCl_2<CaF_2$

B) $CaF_2<CaCl_2<CaBr_2<CaI_2$

C) $CaF_2<CaCl_2<CaI_2<CaBr_2$

D) $CaI_2<CaCl_2<CaBr_2<CaF_2$

Answer: $CaF_2<CaCl_2<CaBr_2<CaI_2$

Solution: Cation is same in all case while anions are different.

According to Fajan's rule, the larger the size of the anion, greater is the covalent character of the ionic bond.

$CaF_2<CaCl_2<CaBr_2<CaI_2$

Q.34. A and B form an ideal solution. If mole fraction of A is 0.3 in liquid phase and vapour pressure of pure A and B is $100\ \text{torr}$ and $150\ \text{torr}$ respectively, then find the mole fraction of A in vapour phase.

A) 0.22

B) 0.33

C) 0.11

D) 0.45

Answer: 0.22

Solution: Assume χ_A and y_A are the mole fractions in liquid and vapour phase respectively.

$\chi_A = 0.3$

$P_T = P_A^O \chi_A + P_B^O \chi_B$

$= 100\,(0.3) + 150\,(0.7)$

$= 30 + 105$

$= 135 \text{ torr}$

$\chi_A \times P_A^O = y_A \times P_T$

$0.3 \times 100 = y_A \times 135$

$y_A = \frac{30}{135} = 0.22$

Q.35. Which of the following product is formed when potassium permanganate is reacted with H_2O_2 in acidic medium?

A) Mn^{4+} and H_2O only

B) Mn^{2+} and H_2O only

C) Mn^{2+}, O_2 and H_2O only

D) Mn^{4+}, O_2 and H_2O only

Answer: Mn^{2+}, O_2 and H_2O only

Solution: In this reaction MnO_4^- act as an oxidising agent and hydrogen peroxide acts as a reducing agent. The reaction is shown below,

$2\,MnO_4^- + 6H^+ + 5H_2O_2 \rightarrow 2\,Mn^{2+} + 5O_2 + 8H_2O$

Q.36. AB_2 dissociates with $t_{\frac{1}{2}} = 200 \text{ sec}$ and the half life remain same irrespective of the initial concentration. Find the time taken in sec for 80% completion of reaction. (Round off to nearest integer) [Given: $\log\ 5 = 0.7,\ \log\ 2 = 0.3$]

A) 467

B) 233

C) 932

D) 117

Answer: 467

Solution: Since half life remain constant throughout, it's a first order reaction

$k = \frac{\ln\ 2}{t_{\frac{1}{2}}}$

$kt = 2.303\ \log \frac{A_0}{A}$

$\frac{\ln\ 2}{t_{\frac{1}{2}}} t = 2.303\ \log \frac{A_0}{A}$

$\frac{2.303\ \log\ 2}{200} t = 2.303\ \log \frac{100}{20}$

$t = \frac{200}{0.3} \times \log\ 5 = 466.67 \text{ sec} \approx 467 \text{ sec}$

Q.37. A chemistry teacher tells you to make a solution of $pH\ 8.26$. If you have $0.2\ M$ solution of NH_3 in $1\ L$, how many moles of NH_4Cl will be added to make the required solution?

[Given: pK_b of $NH_3 = 4.74$]

A) 1

B) 2

C) 3

D) 4

Answer: 2

Solution: A mixture of NH_3 and NH_4Cl will form a basic buffer.

pH of final solution $= 8.26$

pOH of final solution $= 5.74$

Let the number of moles of NH_4Cl added to $1\ L$ solution of be x.

Using Henderson's equation.

$$pOH = pK_b + \log \frac{[NH_4Cl]}{[NH_3]}$$

$$5.74 = 4.74 + \log \frac{x}{0.2}$$

$$\therefore\ x = 2$$

Q.38. $CH_3CH_2MgBr + CH_3OH \rightarrow$ Products

Total number of gaseous products formed in the above reaction is:

A) 2

B) 1

C) 3

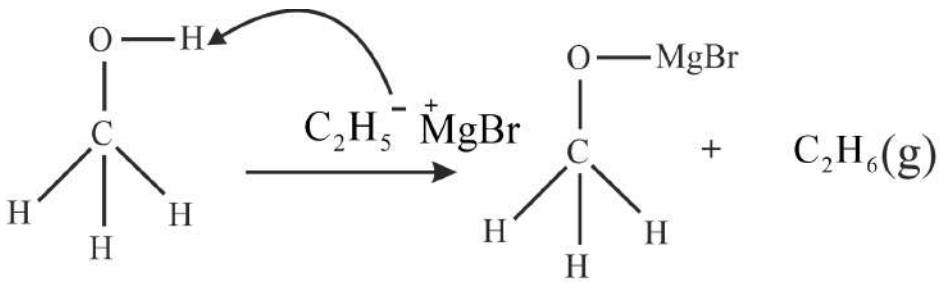
D) 0

Answer: 1

Solution: The reagent ethyl magnesium bromide dissociates as follows:

$$CH_3CH_2MgBr \rightarrow CH_3CH_2^- + Mg^+Br$$

In methanol two types of hydrogens are present. Type-one that are attached to carbon and type-second that are attached to oxygen. The hydrogen attached with oxygen is most acidic and loose as proton, so ethyl nucleophile attacks on hydrogen attached with oxygen in methanol and forms ethane. The negative charge of oxygen is balanced by positively charged magnesium bromide.



Section C: Mathematics

Q.39. If $A=\begin{bmatrix}1\\1\\1\end{bmatrix}, B=\begin{bmatrix}9^2 & 10^2 & 11^2\\12^2 & -13^2 & 14^2\\15^2 & 16^2 & -17^2\end{bmatrix}$, then $A'BA$ (where A' is A transpose) is equal to:

A) $[665]$

B) $[165]$

C) $[765]$

D) $[365]$

Answer: $[665]$

Solution: Given,

$A=\begin{bmatrix}1\\1\\1\end{bmatrix}, B=\begin{bmatrix}9^2 & 10^2 & 11^2\\12^2 & -13^2 & 14^2\\15^2 & 16^2 & -17^2\end{bmatrix}$,

A transpose is given by $A'=[1 \quad 1 \quad 1]$

Now solving, $A'B=\left[9^2+12^2+15^2 \quad 10^2-13^2+16^2 \quad 11^2+14^2-17^2\right]$

Now multiplying with matric A we get, $A'BA=\left[9^2+12^2+15^2 \quad 10^2-13^2+16^2 \quad 11^2+14^2-17^2\right]\begin{bmatrix}1\\1\\1\end{bmatrix}$

$\Rightarrow A'BA=\left[9^2+12^2+15^2+10^2-13^2+16^2+11^2+14^2-17^2\right]$

$\Rightarrow A'BA=[81+144+225+100-169+256+121+196-289]$

$\Rightarrow A'BA=[665]$

Q.40. The value of $\int_0^{20\pi}(|\sin x|+|\cos x|)dx$ is equal to

A) 20

B) 40

C) 60

D) 80

Answer: 80

Solution: Given, $\int_0^{20\pi}(|\sin x|+|\cos x|)dx$

We know that period of $|\sin x|$ & $|\cos x|$ is $\frac{\pi}{2}$

So, $\int_0^{20\pi}(|\sin x|+|\cos x|)dx$

$=40\int_0^{\frac{\pi}{2}}(\sin x+\cos x)dx$

$=40(-\cos x+\sin x)_0^{\frac{\pi}{2}}$

$=40\,(1+1)=80$

Q.41. The total numbers between 1000 and 3000 divisible by 4 using the digits $1,2,3,4,5,6$ will be (repetition of digits not allowed)

A) 30

B) 20

C) 40

D) 10

Answer: 30

Solution: To find total numbers between 1000 and 3000 divisible by 4 using the digits $1, 2, 3, 4, 5, 6,$

We will solve in two cases.

Case I : When first digit is 1.

Then last two digits can be $24, 32, 36, 52, 56$ or 64

So, total ways of choosing last two digit is 6 and second digit will be chosen in 3 ways

So, number of such numbers $= 6 \times 3 = 18$

Case II: When first digit is 2

Then last two digits can be $16, 36, 56$ or 64

So, total ways of choosing last two digit is 4 and second digit will be chosen in 3

So, number of such numbers $= 4 \times 3 = 12$

Total numbers of numbers $= 18 + 12 = 30$

Q.42. The area of region between the curves $y = |x^2 - 1|$ and $y = 1$ is:

A) $\frac{8}{3}\left(\sqrt{2} - 1\right)$

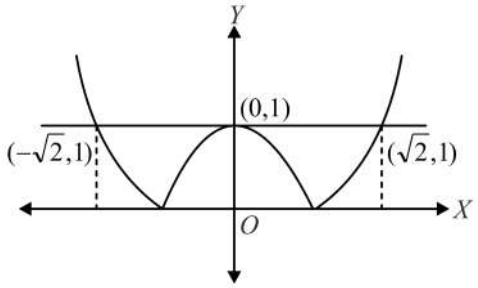
B) $\frac{8}{3}\left(\sqrt{2} + 1\right)$

C) $\frac{4}{3}\left(\sqrt{2} - 1\right)$

D) $\frac{4}{3}\left(\sqrt{2} + 1\right)$

Answer: $\frac{8}{3}\left(\sqrt{2} - 1\right)$

Solution: Plotting the digram of $y = |x^2 - 1|$ and $y = 1$ we get,



Area of the region between the two curves

$= 2\int_0^{\sqrt{2}} \left(1 - |x^2 - 1|\right) dx$

$= 2\left[\int_0^1 \left(1 + \left(x^2 - 1\right)\right) dx + \int_1^{\sqrt{2}} \left(1 - \left(x^2 - 1\right)\right) dx\right]$

$= 2\left[\frac{x^3}{3}\right]_0^1 + 2\left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}$

$= 2\left(\frac{1}{3}\right) + 2\left(2\sqrt{2} - \frac{2\sqrt{2}}{3}\right) - 2\left(2 - \frac{1}{3}\right)$

$= \frac{8}{3}\left(\sqrt{2} - 1\right)$

Q.43. The interval in which abscissa of point P on $y = x^2$ lies such that its distance from $(x - 1)^2 + (y + 1)^2 = 1$ is minimum is

A) $0 < x < \frac{1}{4}$

B) $\frac{1}{4} < x < \frac{1}{2}$

C) $\frac{1}{2} < x < \frac{3}{4}$

D) $\frac{3}{4} < x < 1$

Answer: $\frac{1}{4} < x < \frac{1}{2}$

Solution: Let $P\left(x_1, {x_1}^2\right)$

Minimum distance will be obtained at common normal of the parabola and circle.

Now, the distance of P from given circle,

$d = \sqrt{(x_1 - 1)^2 + \left({x_1}^2 + 1\right)^2} - 1$

For least value of d, we need to minimize

$f(x_1) = (x_1 - 1)^2 + \left({x_1}^2 + 1\right)^2$

i.e. $f'(x_1) = 2(x_1 - 1) + 4x_1\left({x_1}^2 + 1\right) = 0$

From options

$f'\left(\frac{1}{4}\right)$ is $-$ ve and $f'\left(\frac{1}{2}\right)$ is $+$ ve

So, $f'(x_1) = 0$ for some $x_1 \in \left(\frac{1}{4}, \frac{1}{2}\right)$ from IMVT

Q.44. If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, such that m and n are coprime, then $m + n$ is equal to _____ .

A) 166

B) 160

C) 168

D) 163

Answer: 166

Solution: $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \sum_{k=1}^{10} \frac{k}{\left(k^2+k+1\right)\left(k^2-k+1\right)}$

$= \frac{1}{2}\sum_{k=1}^{10}\left(\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right)$

$= \frac{1}{2}\left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{91} - \frac{1}{111}\right)\right]$

$= \frac{1}{2}\left(1 - \frac{1}{111}\right) = \frac{55}{111}$

So $m = 55,\ n = 111$

$\therefore\ \ m + n = 166$

Q.45. The minimum value of the sum of the squares of the roots of the equation $x^2 + (3 - a)x = 2a - 1$ is

A) 6

B) 12

C) 0

D) 16

Answer: 6

Solution: Let α, β be the roots of the equation

$x^2 + (3 - a)x + 1 - 2a = 0$

Then, sum of roots $\alpha + \beta = a - 3$

And product of roots $\alpha\beta = 1 - 2a$

We know that $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\therefore \alpha^2 + \beta^2 = (a - 3)^2 - 2(1 - 2a)$

$= a^2 - 2a + 7$

$= (a - 1)^2 + 6$

$\therefore$ Minimum value of $\alpha^2 + \beta^2 = 6$ at $a = 1$.

Q.46. Let $A = \{1, 2, 3, 4, 5, 6\}, B = \{3, 4, 6, 7, 9\}$ and $C = A \cup B$, then number of elements in cartesian product of $C \times B$ is _____ .

A) 40

B) 20

C) 30

D) 10

Answer: 40

Solution: Given, $A = \{1, 2, 3, 4, 5, 6\}, \quad B = \{3, 4, 6, 7, 9\}$

$C = A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9\}$

Now, number of elements in set C is given by $n(C) = 8$

And number of elements in set B is given by $n(B) = 5$

So, number of elements in $C \times B$ will be $n(C \times B) = 8 \times 5 = 40$

Q.47. Equation of common tangent to parabolas $y = x^2$ and $y = -(x - 2)^2$ is:

A) $y = 4x - 4$

B) $y = -4x + 4$

C) $y = x - 4$

D) $y = x + 4$

Answer: $y = 4x - 4$

Solution: Let $y = mx + c$ be the tangent to $y = x^2$, then

$x^2 - mx - c = 0$

For tangency $D = 0$

$\Rightarrow m^2 = -4c \quad \ldots(1)$

Now let $y = mx + c$ be also tangent to $y = -(x - 2)^2$, then

$(x - 2)^2 + mx + c = 0$

$D = 0$

$\Rightarrow \ m^2 - 8m = 4c \quad \ldots(2)$

From (1) and (2), we get

$m = 0, \ c = 0$ or $m = 4, \ c = -4$

So, there will be two common tangents

$y = 0$ or $y = 4x - 4$

Q.48. If $x - 1 = 0$ is directrix of hyperbola $kx^2 - y^2 = 6$, then the point lying on hyperbola is:

A) $\left(2\sqrt{5}, 6\right)$

B) $\left(\sqrt{5}, -2\right)$

C) $\left(-\sqrt{5}, 3\right)$

D) $\left(-2\sqrt{5}, 3\sqrt{6}\right)$

Answer: $\left(\sqrt{5}, -2\right)$

Solution:

Given equation of hyperbola is $\frac{x^2}{\frac{6}{k}} - \frac{y^2}{6} = 1$

Equation of directrix will be $x = \pm\frac{a}{e}$,

Where $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{6}{\frac{6}{k}}} = \sqrt{1+k}$

$\Rightarrow x = \pm\frac{\sqrt{6}}{\sqrt{k}(\sqrt{1+k})}$

i.e. $\pm\frac{\sqrt{6}}{\sqrt{k}(\sqrt{1+k})} = 1$

$\therefore \quad \left(\sqrt{k}\sqrt{1+k}\right)^2 = 6 \Rightarrow k^2 + k - 6 = 0$

$\Rightarrow \ k = 2$ as $k = -3$ is rejected

So equation of hyperbola will be $H \equiv \frac{x^2}{3} - \frac{y^2}{6} = 1$

$\therefore \quad \left(\sqrt{5}, -2\right)$ satisfy the given hyperbola

Q.49. If $z = x + iy$, $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then which of the following is TRUE

A) $x^2 + 2y + 4 = 0$

B) $x^2 - 2y + 4 = 0$

C) $x + y = 0$

D) $x^2 - y + 4 = 0$

Answer: $x^2 + 2y + 4 = 0$

Solution:

Given, $z = x + iy$

Also, $|z| = 2$

So, $x^2 + y^2 = 4 \quad \ldots(1)$

Now, $|z - i| = |z + 5i|$

$\Rightarrow |z - i|^2 = |z + 5i|^2$

$\Rightarrow x^2 + (y-1)^2 = x^2 + (y+5)^2$

$\Rightarrow y = -2 \quad \ldots(2)$

So, $x = 0$ by solving equation (1) & (2)

Hence, only $x^2 + 2y + 4 = 0$ is true, as point only satisfy this equation.

Q.50. $\lim\limits_{x\to 0}\frac{\alpha x - \left(e^{3x} - 1\right)}{\alpha x\left(e^{3x} - 1\right)} = \beta$, then $\alpha + \beta$ is equal to:

A) $\frac{5}{2}$

B) $\frac{7}{2}$

C) 1

D) 2

Answer: $\frac{5}{2}$

Solution:

Given, $\lim_{x\to 0}\frac{\alpha x-\left(e^{3x}-1\right)}{\alpha x\left(e^{3x}-1\right)}=\beta$

$\Rightarrow \lim_{x\to 0}\frac{\alpha x+1-e^{3x}}{\frac{\alpha x\left(e^{3x}-1\right)}{3x}\cdot 3x}=\beta$

$\Rightarrow \lim_{x\to 0}\frac{\alpha x+1-e^{3x}}{3\alpha x^2}=\beta$ as $\left(\lim_{x\to o}\frac{e^x-1}{x}=1\right)$

$\Rightarrow \lim_{x\to 0}\frac{\alpha-3e^{3x}}{6\alpha x}=\beta$ [Using L' Hospital rule] as $\left(\frac{0}{0}\text{ form}\right)$

Now for limit to exist, $\alpha=3$

So, $\lim_{x\to 0}\frac{3-3e^{3x}}{6\times 3\times x}=\beta$

So, $\beta=\lim_{x\to 0}\frac{-\left(e^{3x}-1\right)}{6x}=-\frac{1}{2}$

$\therefore\ \alpha+\beta=\frac{5}{2}$

Q.51. Biased coins have probability of getting head is $\frac{2}{3}$ and x is number of heads when six coins are tossed, then the probability $P(X\le 2)$ is equal to:

A) $\frac{73}{729}$

B) $\frac{67}{729}$

C) $\frac{23}{729}$

D) $\frac{73}{243}$

Answer: $\frac{73}{729}$

Solution:

Let p is the probability of getting head, so $p=\frac{2}{3}$

So, $q=1-p=1-\frac{2}{3}=\frac{1}{3}$ and given $n=6$

We know that, $P(X\le 2)={}^6C_0(p)^0(q)^6+{}^6C_1(p)^1(q)^5+{}^6C_2(p)^2(q)^4$

$\Rightarrow P(X\le 2)={}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6+{}^6C_1\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5+{}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4$

$\Rightarrow P(X\le 2)=\frac{1}{3^6}+\frac{12}{3^6}+\frac{60}{3^6}$

$\Rightarrow P(X\le 2)=\frac{73}{3^6}=\frac{73}{729}$

Q.52. Let roots of $x^2-4x-6=0$ are the abscissa and roots of $y^2+2y-7=0$ are the ordinates of the end of diameter of the circle $x^2+y^2+2ax+2by+c=0$ then $a+b-c$ is equal to

A) 10

B) 11

C) 12

D) -12

Answer: 12

Solution: Given that the roots of $x^2-4x-6=0$ are the abscissa of the end of diameter

i.e. $x_1+x_2=4,\ x_1x_2=-6$

and roots of $y^2+2y-7=0$ are ordinate of the end of diameter

i.e. $y_1+y_2=-2,\ y_1y_2=-7$

Now, equation of the circle will be

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

i.e. $x^2-(x_1+x_2)x+x_1x_2+y^2-(y_1+y_2)y+y_1y_2=0$

$\Rightarrow x^2+y^2-4x+2y-13=0$

$\therefore a=-2,\ \ b=1,\ \ c=-13$

$\Rightarrow a+b-c=-2+1+13=12$

Q.53. Let $A=\{1,2,3,\cdots,7\}$ and $B=\{3,6,7,9\}$, then the number of subsets C of A such that $C\cap B\neq\phi$ is

Answer: 112

Solution: Number of subsets of $A=2^7=128$

If $C\cap B=\phi$ then set C must contain only $1,2,4$ or 5

Number of such subsets $C=2^4=16$

Hence, the number of required subsets $=128-16=112$