JEE Main Exam 2022 - Session 2

27 Jul 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. A DC current of $4\ \mathrm{A}$ and AC current of peak value $4\ \mathrm{A}$ passes through $3\ \Omega$ and $2\ \Omega$ resistors respectively. Find the ratio of heat generated

A) $3:1$

B) $3:2$

C) $3:4$

D) $1:1$

Answer: $3:1$

Solution: Heat in DC is given by,

$H_{DC}=\left(i^2R\right)t=4^2\times 3\times t=48t$

Heat in AC is given by,

$H_{AC}=(V_{\mathrm{rms}}\ I_{\mathrm{rms}}\cos\phi)t$

Since, only resistance is present $Z=R\Rightarrow V_{rms}=I_{rms}R$ and $\cos\phi=1$

$\Rightarrow H_{AC}=(I_{\mathrm{rms}}\ )^2R\cos\phi t=I_{\mathrm{rms}}^2\ Rt=\left(\frac{4}{\sqrt{2}}\right)^2\times 2\times t$

$\Rightarrow H_{AC}=16t$

Therefore, the ratio $\frac{H_{DC}}{H_{AC}}=\frac{3}{1}$

Q.2. In a station, the TV transmission tower is of height $100\ \mathrm{m}$. In order to triple the coverage range, the height should be increased to

A) $200\ \mathrm{m}$

B) $300\ \mathrm{m}$

C) $600\ \mathrm{m}$

D) $900\ \mathrm{m}$

Answer: $900\ \mathrm{m}$

Solution: Range of signal from an antenna is given by,

$r=\sqrt{2hR}$

Therefore,

$\frac{h_2}{h_1}=\left(\frac{r_2}{r_1}\right)^2$

$\Rightarrow h_2=3^2\times 100=900\ \mathrm{m}$

Q.3. Two bar magnets oscillate in earth magnetic field with the time period of $3\ \mathrm{s}$ and $4\ \mathrm{s}$. If ratio of moment of inertia is $3:2$, then the ratio of the magnetic moment is

A) $4:1$

B) $8:3$

C) $27:16$

D) $2:1$

Answer: $8:3$

Solution: The time period of bar magnets is given by,

$$T=2\pi\sqrt{\frac{I}{MB}} \Rightarrow M \propto \frac{I}{T^2}$$

$$\Rightarrow \frac{M_1}{M_2}=\frac{I_1}{I_2}\times\left(\frac{T_2}{T_1}\right)^2$$

$$\Rightarrow \frac{M_1}{M_2}=\frac{3}{2}\times\left(\frac{4}{3}\right)^2=\frac{8}{3}$$

Q.4. Maximum error in the measurement of force is 5% and that in measurement of length is 5%. Maximum possible error in measurement of torque is (angle between force and position vector is known to be accurate)

A) 10%

B) 20%

C) 15%

D) 5%

Answer: 10%

Solution: Torque of a force is given by,

$$\vec{\tau}=\vec{r}\times\vec{F}$$

$$\Rightarrow \tau=rF\sin\theta$$

As the angle is known to be accurate,

$$\Rightarrow \frac{\Delta\tau}{\tau}\times 100=\left[\frac{\Delta r}{r}+\frac{\Delta F}{F}\right]\times 100$$

Therefore, percentage error in $\tau=5\%+5\%$

$=10\%$

Q.5. A bag is dropped on a conveyor belt with coefficient of friction μ. The belt keeps on moving with constant speed v. The time it takes for relative motion between bag and belt to stop is

A) $\frac{v}{g}$

B) $\frac{v}{2\mu g}$

C) $\frac{v}{\mu g}$

D) $\frac{g}{v}$

Answer: $\frac{v}{\mu g}$

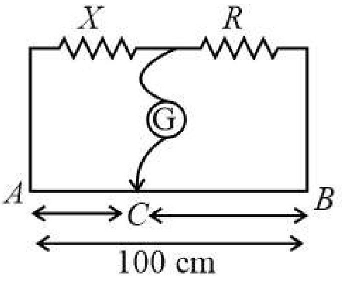
Solution: The acceleration of the bag when it is dropped on the conveyor belt will be,

$a=\mu g.$

The relative velocity between the bag and the conveyor will stop when the bag will achieve velocity of v. Using the equation of motion,

$$v=u+at \Rightarrow v=\mu gt$$
$$\Rightarrow t=\frac{v}{\mu g}$$

Q.6. In the shown meter bridge the null point C is obtained $30\ \text{cm}$ away from point A. If $R = 5.6\ \text{k}\Omega$, then the value of resistance of resistor X is



A) $1.6\ \text{k}\Omega$

B) $2.4\ \text{k}\Omega$

C) $4.8\ \text{k}\Omega$

D) $5.8\ \text{k}\Omega$

Answer: $2.4\ \text{k}\Omega$

Solution: For the null point of the potentiometer,

$\frac{X}{R} = \frac{l}{L-l}$

$\Rightarrow \frac{X}{5.6} = \frac{30}{70}$

$\Rightarrow X = 3 \times 0.8 = 2.4\ \text{k}\Omega$

Q.7. Waves of intensity I and $4I$ with same frequency interfere at two points A and B such that at A, phase difference between the two is $\frac{\pi}{2}$, while at B it is $\frac{\pi}{3}$. The difference between net intensities at two points is equal to

A) I

B) $7I$

C) $3I$

D) $2I$

Answer: $2I$

Solution: The intensity of the two waves interfering at a point having phase difference ϕ is given by,

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$

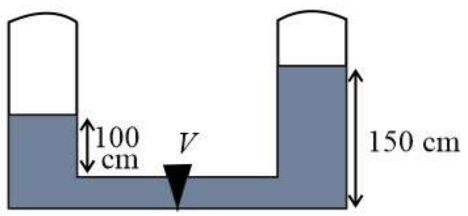
$\Rightarrow I_A = I + 4I + 2\sqrt{4I^2} \times \cos \frac{\pi}{2} = 5I$

and

$I_B = I + 4I + 2\sqrt{4I^2} \times \cos \frac{\pi}{3} = 5I + 2I = 7I$

$\Delta I = |I_A - I_B| = 2I$

Q.8. The cylinders shown have an area of cross-section of $16\ \text{cm}^2$. In one cylinder the water is raised upto $100\ \text{cm}$ while in other the water is raised upto $150\ \text{cm}$. After opening of the valve V, the work done by gravity when levels set is equal to



A) $5\ \text{J}$

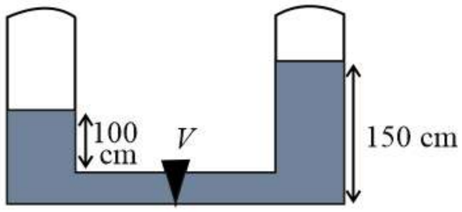
B) $1\ \mathrm{J}$

C) $17.25\ \mathrm{J}$

D) $25\ \mathrm{J}$

Answer: $1\ \mathrm{J}$

Solution:



Initially, the centre of mass of the right column will be at height of $75\ \mathrm{cm}$ and the centre of mass of the left column will be at $50\ \mathrm{cm}$. After opening of the valve both columns will have height of $125\ \mathrm{cm}$ and their centre of mass will be at height $\frac{125}{2}\ \mathrm{cm}$. Therefore,

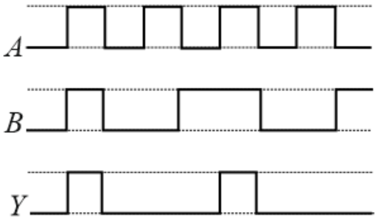
$$W=-\Delta U=-\left[(m_1+m_2)g\frac{h_1+h_2}{4}-m_1g\frac{h_1}{2}-m_2g\frac{h_2}{2}\right]$$

Now, $m_1=\rho A h_1$ and $m_2=\rho A h_2$

Putting $\rho=1000\ \mathrm{kg\ m^{-3}}$, $g=10\ \mathrm{m\ s^{-2}}$, $h_1=100\ \mathrm{cm}$, $h_2=150\ \mathrm{cm}$ and $A=16\times10^{-4}\ \mathrm{m^2}$, we get,

$W=1\ \mathrm{J}$

Q.9. The shown signals A and B are used as input to logic gate with Y as its output as shown. Identify the logic gate.



A) AND

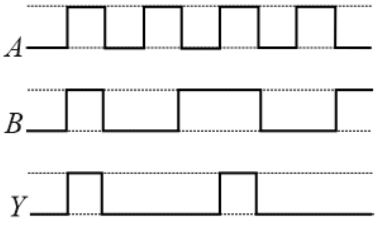
B) OR

C) NOR

D) NAND

Answer: AND

Solution:



From the above diagram, we can see,

if $A=0,\ B=0$, then $Y=0$.

if $A=1,\ B=0$, then $Y=0$.

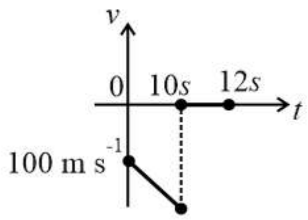
if $A=0,\ B=1$, then $Y=0$.

if $A=1,\ B=1$, then $Y=1$.

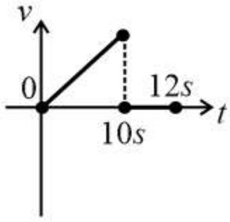
Clearly, the logic gate is a AND gate.

Q.10. A bullet is shot vertically downwards at $100 \mathrm{~m~s}^{-1}$ and strikes ground below after $10 \mathrm{~s}$ then it stays at rest there for further $2 \mathrm{~s}$. The appropriate $v-t$ graph will be,

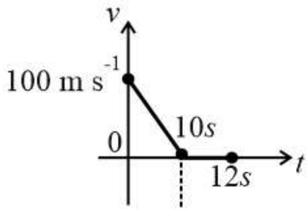
A)



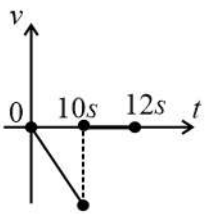
B)



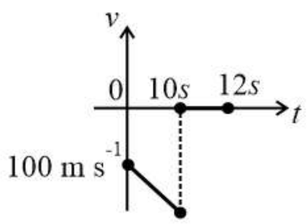
C)



D)



Answer:



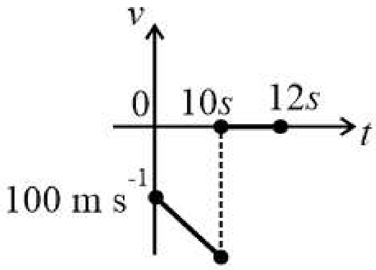
Solution: Let velocity of ball at any time is v during the downward motion, then we can write,

$v=u+gt$
$\Rightarrow v=-100-10t$

Velocity at the end of $t=10\ \text{s}$, $v=-200\ \text{m s}^{-1}$

Once bullet strikes ground, it is at rest and hence velocity for that duration is $0\ \text{m s}^{-1}$.

Hence, upto $10\ \text{s}$ velocity of the ball will decrease and the graph will be straight line with negative slope and from $10\ \text{s}$ onwards, it will be a horizontal line on X-axis.



Q.11. Two satellites A and B have mass ratio $1:3$ and their distance from the centre of earth are $3r$ and $4r$ respectively. Their total energies are in the ratio. (neglecting self energy)

A) $\frac{4}{9}$

B) $\frac{16}{9}$

C) $\frac{9}{4}$

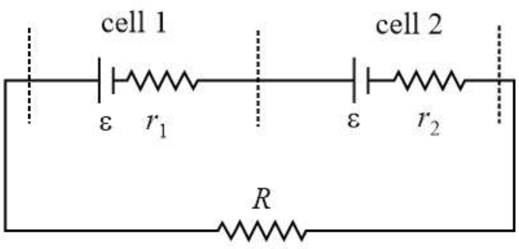
D) $\frac{9}{16}$

Answer: $\frac{4}{9}$

Solution: Total energy of the satellite at r distance from the centre of the earth is given by, $TE=\frac{-GMm}{2r}$.

Now, we can write, $\Rightarrow \frac{TE_A}{TE_B}=\frac{\frac{-GM(m)}{2(3r)}}{\frac{-GM(3m)}{2(4r)}}=\frac{4}{9}$

Q.12. Two cells, cell 1 and cell 2 are in series across the resistance R. Find the value of R such that potential difference across cell 1 is zero.



A) r_1-r_2

B) r_1+r_2

C) $\frac{(r_1r_2)}{r_1+r_2}$

D) $\frac{r_1+r_2}{2}$

Answer: $r_1 - r_2$

Solution: Current through the wire will be,

$I = \frac{2\varepsilon}{R+r_1+r_2}$.

Now potential difference across cell 1 will be, $\varepsilon - Ir_1 = 0$

$\Rightarrow \varepsilon = \frac{2\varepsilon}{R+r_1+r_2} \times r_1$
$\Rightarrow R + r_1 + r_2 = 2r_1$
$\Rightarrow R = r_1 - r_2$

Q.13. Two point charges each of magnitude Q are kept fixed at a distance $2d$ from each other. A third charge q is kept at middle of the two charges and is displaced a little along the line joining the two charges. The time period of oscillations is (Take mass $m_q = m$)

A) $2\pi\sqrt{\frac{\pi\epsilon_0 md^3}{Qq}}$

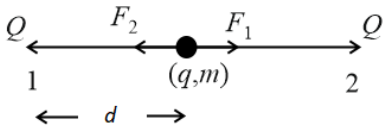
B) $\pi\sqrt{\frac{\pi\epsilon_0 md^3}{Qq}}$

C) $2\pi\sqrt{\frac{\pi\epsilon_0 md}{Qq}}$

D) $2\sqrt{\frac{\pi\epsilon_0 md^3}{Qq}}$

Answer: $2\pi\sqrt{\frac{\pi\epsilon_0 md^3}{Qq}}$

Solution:



Let the charged particle be displaced slightly towards right by distance $x\,(x \ll d)$.

$(Q\ at\ A,\ q): F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(d+x)^2}$, towards right

$(Q \text{ at } B,\ q):\ F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(d-x)^2}$, towards left

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$, will bring it towards $O.\vec{F}_{net} = \frac{1}{4\pi\epsilon_0}\left[\frac{1}{(d+x)^2} - \frac{1}{(d-x)^2}\right]\hat{i}$

$= \frac{-Qq}{4\pi\epsilon_0}\left[\frac{4xd}{\left(d^2-x^2\right)^2}\right]\hat{i}$

Since $x \ll d,\ x^2$ is negligible.

$\vec{F}_{net} = -\left(\frac{Qq}{\pi\epsilon_0 d^3}\right) \times x\hat{i}$

Clearly, direction of the force is opposite to the displacement & $F \propto x$, hence it is a SHM.

Acceleration of charged particle is given as,

$a = \frac{F}{m} = -\frac{Qq}{\pi\epsilon_0 m d^3}x \quad \ldots.(1)$

and we know that,

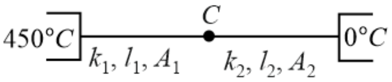
$a = -\omega^2 x \quad \ldots.(2)$

Comparing (1) and (2), we get

$\omega = \sqrt{\frac{Qq}{\pi\epsilon_0 m d^3}}$

Hence, time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\pi\epsilon_0 m d^3}{Qq}}$

Q.14. Two rods, rod 1 and rod 2 are connected in series with each other in between two reservoirs maintained at $450\ ^\circ\mathrm{C}$ and $0\ ^\circ\mathrm{C}$ as shown. If $\frac{k_1}{k_2} = 9,\ \frac{l_1}{l_2} = 2$ and $\frac{A_1}{A_2} = 2$, then temperature of point C is (k is thermal conductance)



A) $350\ ^\circ\mathrm{C}$

B) $45\ ^\circ\mathrm{C}$

C) $400\ ^\circ\mathrm{C}$

D) $405\ ^\circ\mathrm{C}$

Answer: $405\ ^\circ\mathrm{C}$

Solution: Let temperature at the point C is T. In series connection, heat current will be same through both the wires. Then,

$$\frac{k_1A_1(450^\circ - T)}{l_1} = \frac{k_2A_2(T - 0^\circ)}{l_2}$$
$$\Rightarrow \left(\frac{k_1}{k_2}\right)\left(\frac{A_1}{A_2}\right)(450^\circ - T) = T\left(\frac{l_1}{l_2}\right)$$
$$\Rightarrow 4050 - 9T = T$$
$$\Rightarrow T = \frac{4050}{10} = 405\ ^\circ\text{C}$$

Q.15. A radioactive substance takes 30 years to reduce to $\left(\frac{1}{16}\right)^{th}$ of its initial concentration. Find half life of the substance(in years).

A) 7.5

B) 15

C) 5

D) 7

Answer: 7.5

Solution:

As we know, $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}}$.

Given, $t = 30$ years for $\frac{N}{N_0} = \frac{1}{16}$.

Therefore,

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{30}{t_{\frac{1}{2}}}}$$
$$\Rightarrow \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{30}{t_{\frac{1}{2}}}}$$
$$\Rightarrow 4 = \frac{30}{t_{\frac{1}{2}}}$$
$$\Rightarrow t_{\frac{1}{2}} = 7.5 \text{ years}$$

Section B: Chemistry

Q.16. The elevation in boiling point of a 2% aqueous solution of nonvolatile solute A is equal to the boiling point of 3% aqueous solution of non volatile solute B. Find the relation between their molecular weight. (Consider both solutes are non electrolytes.)

A) $M_A = 4M_B$

B) $M_B = 4M_A$

C) $3M_A = 2M_B$

D) $3M_B = 2M_A$

Answer: $3M_A = 2M_B$

Solution: $(\Delta T_b)_A = (\Delta T_b)_B$

$k_b m_A = k_b m_B$

$\left[\frac{(W_A)}{M_A \times W_{solvent}}\right] = \left[\frac{W_B}{M_B \times W_{solvent}}\right]$

$\frac{2}{M_A \times 98} = \frac{3}{M_B \times 97}$

$2M_B = 3M_A$

Q.17. Find out the solubility product of CaF_2 if solubility of CaF_2 is $2.34\ g/100\ mL$.

A) $0.108\ (mol/L)^3$

B) $0.072\ (mol/L)^3$

C) $0.036\ (mol/L)^3$

D) $0.032\ (mol/L)^3$

Answer: $0.108\ (mol/L)^3$

Solution: $W_B = 2.34\ g$

$M_B = 40 + 19 + 19 = 78\ g/mol$

$CaF_2(s) \Leftrightarrow \underset{s}{Ca^{2+}}(aq) + \underset{2s}{2F^-}(aq)$

$\underset{(Solubility)}{s}$ $\rightarrow$"Molarity"

$k_{SP} = s \times (2s)^2$

$= 4s^3$

$M = \frac{2.34}{78} \times \frac{1000}{100} = 0.3M$

$4s^3 = 4 \times (0.3)^3$

$= 4 \times 27 \times 10^{-3}$

$= 108 \times 10^{-3}$

$= 0.108 (mol/L)^3$

Q.18. Assertion: SO_2 is adsorbed more easily than CH_4.

Reason: Molar mass of SO_2 is greater than CH_4 and SO_2 has a non-zero dipole moment.

A) Both assertion and reason are correct and reason is correct explanation of assertion

B) Both assertion and reason are correct and reason is not correct explanation of assertion

C) Assertion is correct but reason is incorrect

D) Assertion is incorrect but reason is correct

Answer: Both assertion and reason are correct and reason is correct explanation of assertion

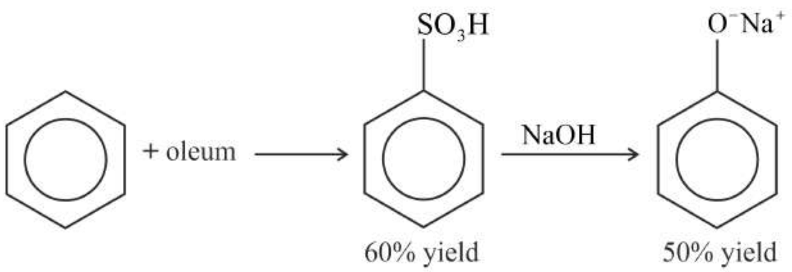
Solution: Lesser the value of critical temperature of gases lesser will be the extent of adsorption.

Gas	SO_2	CH_4
Critical temperature/K	630	190

The critical temperature depends on the strength of the intermolecular interactions that hold a substance together as a liquid.

SO_2 has stronger intermolecular force of attractions than CH_4, due to the more molar mass and non-zero dipole moment.

Q.19. The percentage yield of the complete reaction is:



A) 50%

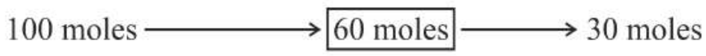
B) 60%

C) 30%

D) 100%

Answer: 30%

Solution: Given that 60% yield in the first step, i.e, 100 moles of reactant gives 60 moles of the product. In the second step 50% yiled is given, hence, 60 moles of benzene sulphonic acid gives 30 moles of phenoxide.



% yield $= 30\%$

Q.20. Match List-I with List-II.

	List 1 (Polymer)		List 2 (Commercial name)
(A)	Phenol formaldehyde resin	(1)	Glyptal
(B)	Copolymer of 1, 3-butadiene and styrene	(2)	Novolac
(C)	Polymer of glycol and phthalic acid	(3)	Buna-S
(D)	Polymer of glycol and terephthalic acid	(4)	Dacron

A) (A)-(1), (B)-(3), (C)-(2), (D)-(4)

B) (A)-(2), (B)-(3), (C)-(1), (D)-(4)

C) (A)-(4), (B)-(2), (C)-(3), (D)-(1)

D) (A)-(3), (B)-(4), (C)-(2), (D)-(1)

Answer: (A)-(2), (B)-(3), (C)-(1), (D)-(4)

Solution: Novolacs are phenol-formaldehyde resins with a formaldehyde to phenol molar ratio of less than one.

The rubber obtained is also called Styrene butadiene rubber (SBR). In Buna-S, Bu stands for butadiene and, Na for sodium and S for styrene. It is vulcanised with sulfur. The rubber is slightly inferior to natural rubber in its physical properties.

Glyptal- a polyester is formed by ethylene glycol and phthalic acid by step-growth polymerization.

Dacron obtained by condensation polymerisation of ethylene glycol and terephthalic acid at $420 - 460$ K using zinc acetate and antimony oxide as catalyst.

Q.21. Find out the number of paramagnetic species:

$KO_2,\ Na_2O,\ NO,\ NO_2,\ N_2O_3,\ Cl_2O,\ SO_2,\ SO_3$

A) 2

B) 3

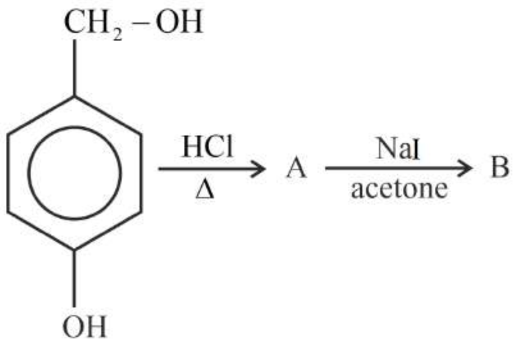
C) 4

D) 5

Answer: 3

Solution: A molecule with odd electrons have one or more unpaired electrons. Molecules with unpaired electrons are para-magnetic. If all electrons are paired, then molecule is diamagnetic. $KO_2,\ NO$ and NO_2 are paramgnetic substances.

Q.22. The product B is



A)

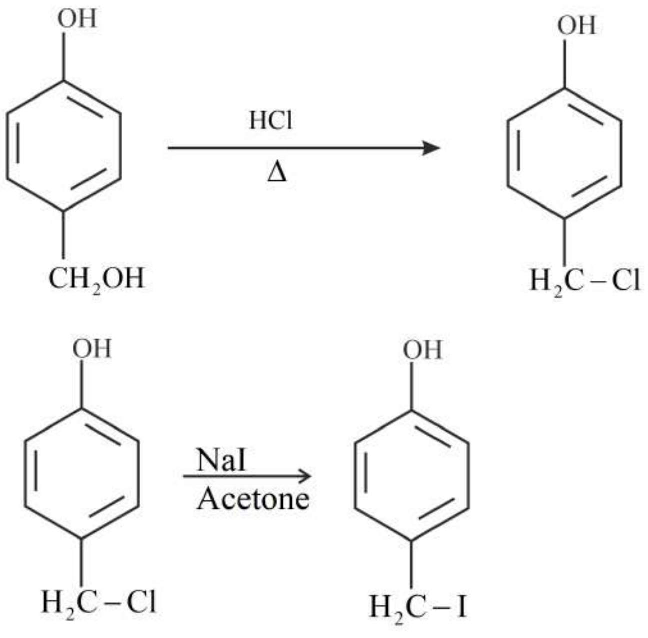
B)

C)

D)

Answer:

Solution: Alcohols are more reactive than phenol towards the reaction with hydrogen chloride. The preparation of alkyl iodide from alkyl bromide or chloride with potassium or sodium iodide in acetone is generally known as the Finkelstein reaction.



Q.23. In the titration of $KMnO_4$ with oxalic acid in acidic medium, the change in oxidation number of carbon per molecule of oxalic acid is

A) 4

B) 3

C) 2

D) 1

Answer: 2

Solution: Potassium permanganate is standardized against pure oxalic acid. It involves a redox reaction. Oxalic acid is oxidised to carbon dioxide by $KMnO_4$, which itself gets reduced to $MnSO_4$. Oxalic acid reacts with potassium permanganate in the following way.

$$2KMnO_4 + 3H_2SO_4 + 5(COOH)_2 \rightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 10CO_2\uparrow$$

The change in oxidation state of carbon per molecule of oxalic acid is 2.

Q.24. Which oxoacid has maximum oxygen atoms?

A) Hypophosphorus acid

B) Ortho phosphoric acid

C) Pyrophosphoric acid

D) Phosphonic acid

Answer: Pyrophosphoric acid

Solution:

Hypophosphorous (Phosphinic) acid	H_3PO_2
Orthophosphorous (Phosphonic) acid	H_3PO_3
Orthophosphoric acid	H_3PO_4
Pyrophosphoric acid	$H_4P_2O_7$

Q.25. Find out the number of species having similar bond order.

CN^-, O_2^{2-}, O_2^{2+}, O_2^-, NO

A) 2

B) 3

C) 4

D) 5

Answer: 2

Solution:

Number of e^-	Bond order
1	0.5
2	1
3	0.5
4	0
5	0.5
6	1
7	0.5
8	0
9	0.5
10	1
11	1.5
12	2
13	2.5
14	3
15	2.5
16	2
17	1.5
18	1
19	0.5
20	0

CN^- and O_2^{2+} are having 14 electrons each, hence the bond order is 3.

Q.26. Consider the following statements:

Statement I: $KMnO_4$, $K_2Cr_2O_7$ and Fe^{3+} can convert I^- to I_2 independently.

Statement II: Manganate ion is paramagnetic.

A) Statement I and II both correct

B) Statement I is correct and statement II is incorrect

C) Statement I is incorrect and statement II is correct

D) Both statement I and II are incorrect

Answer: Statement I and II both correct

Solution: Iodine is liberated from potassium iodide.

$$10I^{-} + 2MnO_4^{-} + 16H^{+} \rightarrow 2Mn^{2+} + 8H_2O + 5I_2$$

$$\underset{\text{Potassium dichromate}}{K_2Cr_2O_7} + 7H_2SO_4 + 6KI \rightarrow Cr_2(SO_4)_3 + 3I_2 + 4K_2SO_4 + 7H_2O$$

$$2Fe^{3+} + 2KI \rightarrow 2Fe^{2+} + I_2 + 2K^{+}$$

Manganate ion is paramagnetic in nature with one unpaired electron.

Q.27. 250 g of an aqueous solution contains D-glucose. The % of carbon (by mass) present in the solution is 16.7. Find the molality of the solution.

A) 2.56 m

B) 3.98 m

C) 3.24 m

D) 1.78 m

Answer: 3.98 m

Solution: 1. D-glucose $C_6H_{12}O_6$

2. Molar Mass : 180 g

3.
C_6
$\downarrow H_{12}O_6$
72 g of C is 180 g of glucose

Mass of C $= \frac{16.7}{100} \times 250 = 41.75$ g

Mass of D-glucose $= \frac{180}{72} \times 41.75 = 104.375$ g

Mass of solvent $= 145.625$ g

$$\text{molality} = \frac{\text{mass of D-glucose}}{\text{Molar mass of glucose}} \times \frac{1000}{\text{Mass of solvent}}$$
$$= \frac{104.375}{180} \times \frac{1000}{145.625} = 3.98 \text{ m}$$

Q.28. Consider the following statements:

Statement I: The energy of electron in $2s$ orbital of He^{+} is more than that of Li^{2+}

Statement II: The energy of an electron in an orbital decreases with increase atomic number.

A) Both statements are correct and statement II is correct explanation of statement I

B) Both statements are correct and statement II is not correct explanation of statement I

C) Statement I is correct and statement II is incorrect

D) Statement I is incorrect and statement II is correct

Answer: Both statements are correct and statement II is correct explanation of statement I

Solution: Energy of electron in n^{th} orbit of a Bohr atom is, $E_n = -13.6\frac{z^2}{n^2}eV$

Greater the value of atomic number, lesser is the energy.

Q.29. Match List-I consisting of pollutant to List-II consisting of disease

List I		List II	
A.	Sulphate > 500 ppm	P.	Methemoglobinemia
B.	Nitrate > 50 ppm	Q.	Mottling of teeth
C.	Lead > 50 ppb	R.	Laxative effect
D.	Fluoride > 2 ppm	S.	Kidney damage

A) A-P, B-Q, C-R, D-S

B) A-R, B-P, C-S, D-Q

C) A-Q, B-R, C-P, D-S

D) A-Q, B-R, C-S, D-P

Answer: A-R, B-P, C-S, D-Q

Solution: F^- ion concentration above 2 ppm causes brown mottling of teeth. At the same time, excess fluoride (over 10 ppm) causes harmful effect to bones and teeth

Lead: Drinking water gets contaminated with lead when lead pipes are used for transportation of water. The prescribed upper limit concentration of lead in drinking water is about 50 ppb. Lead can damage kidney, liver, reproductive system etc.

Sulphate: Excessive sulphate (> 500 ppm) in drinking water causes laxative effect, otherwise at moderate levels it is harmless.

Nitrate: The maximum limit of nitrate in drinking water is 50 ppm. Excess nitrate in drinking water can cause disease such as methaemoglobinaemia ('blue baby' syndrome).

Q.30. 10 mL of Fe^{2+} ion react completely with 20 mL of 0.02 molar acidified $K_2Cr_2O_7$ solution. Find the molarity of Fe^{2+} ion.

A) 0.12

B) 0.24

C) 0.36

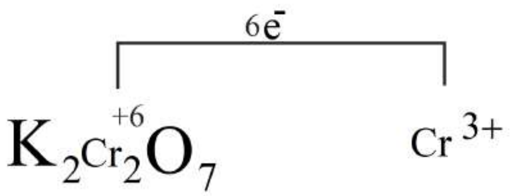
D) 0.48

Answer: 0.24

Solution:

$$Fe^{2+} \longrightarrow Fe^{3+}$$

n-factor for above equation $= 1$



$n_f = 6$

$(M \times n_f \times V)_{Fe^{2+}} = (M \times n_f \times V)_{K_2Cr_2O_7}$

$M \times 10 \times 1 = 0.02 \times 20 \times 6$

$M = 0.24$ M

Q.31. Number of basic oxides among the following compounds is:

$NO_2,\ N_2O,\ NO,\ Li_2O,\ SO_2,\ PbO$

A) 4

B) 3

C) 2

D) 1

Answer: 1

Solution: NO_2, SO_2 are acidic oxides

N_2O, NO, are neutral oxides

Li_2O is basic oxide

PbO is an amphoteric oxide

Q.32. Assertion: Hydrogen peroxide can act both as an oxidizing agent as well as reducing agent in acidic as well as basic medium.

Reason: Density of H_2O_2 is less than that of D_2O

A) Both assertion and reason are correct

B) Assertion is correct and reason is false

C) Assertion is false and reason is correct

D) Both assertion and reason is false

Answer: Assertion is correct and reason is false

Solution: It acts as an oxidising as well as reducing agent in both acidic and alkaline media. Simple reactions are described below.

(i) Oxidising action in acidic medium

$2Fe^{2+}(aq) + 2H^{+}(aq) + H_2O_2(aq) \rightarrow 2Fe^{3+}(aq) + 2H_2O(l)$

(ii) Reducing action in acidic medium

$2MnO_4^{-} + 6H^{+} + 5H_2O_2 \rightarrow 2Mn^{2+} + 8H_2O + 5O_2$

(iii) Oxidising action in basic medium

$2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^{-}$

(iv) Reducing action in basic medium

$I_2 + H_2O_2 + 2OH^{-} \rightarrow 2I^{-} + 2H_2O + O_2$

The density of hydrogen peroxide is more than that of heavy water

Q.33. Which of the following are not methods of purification for metals?
(A) Poling,
(B) Electrolysis,
(C) Liquation,
(D) Leaching,
(E) Calcination,
(F) Distillation

A) (A), (B) and (C)

B) (B), (D) and (F)

C) (D) and (F)

D) (D) and (E)

Answer: (D) and (E)

Solution: Poling is a method used to purify metals that have oxidized impurities. It is typically used to purify metals like copper or tin that are in the impure form of a copper oxide or tin oxide. A log of wood which is still green is used to stir the liquid metal.

Electrolytic refining is a technique that is used for the extraction and purification of metals that are obtained by refining methods. The impure metal is used as an anode and the pure metal is used a cathode. Soluble salt from the same metal is used an electrolyte.

Liquation process is used for the purification of the metal, which itself is readily fusible, but the impurities present in it are not, used for the purification of Sn and Zn, and for removing Pb from Zn-Aq alloy.

Leaching is a process widely used in extractive metallurgy where ore is treated with chemicals to convert the valuable metals within into soluble salts while the impurity remains insoluble.

Calcination is the process in which the ore of the metal is heated to high temperature in the absence or limited supply of air or oxygen.

Distillation: This method is used for the purification of metals which possess a low boiling point such as mercury and zinc.

Section C: Mathematics

Q.34. Let $a_1, a_2, a_3, \cdots, a_n$ be in A.P. The ratio of sum of first five terms to the sum of first nine terms is $5:17$. Also $110 < a_{15} < 120$. Find the sum of first 10 terms of the A.P (where all $a_i\,(i = 1, 2, 3, \cdots, n)$ are integers)

A) 330

B) 460

C) 290

D) 380

Answer: 380

Solution: Let the first term be a and common difference be d.

Given $\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)} = \frac{5}{17}$

$\Rightarrow d = 4a$

Also given $110 < a_{15} < 120 \Rightarrow a + 14d \in (110, 120) \Rightarrow a = 2$

so $d = 8$

Now $S_{10} = \frac{10}{2}(4 + 9\,(8)) = 380$

Q.35. Let S be sample space for 5 digit numbers. If p is probability of a number being randomly selected which is multiple of 7 but not divisible by 5, then $9p$ is equal to:

A) 1.0146

B) 1.2085

C) 1.0285

D) 1.1521

Answer: 1.0285

Solution: We know that five-digit numbers lie from 10000 to 99999

$\therefore n\,(S) = 90000$

Now the number of numbers which are divisible by $7 = \frac{90000}{7}$

Similarly, the number of numbers which are divisible by 7 and $5 = \frac{90000}{35}$

$\therefore$ Required probability, $p = \frac{\frac{90000}{7} - \frac{90000}{35}}{90000}$

$\Rightarrow \quad p = \frac{4}{35}$

i.e. $9p = 1.0285$

Q.36. Let $A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$, $\alpha,\ \beta \in R$ such that $\alpha A^2 + \beta A = 2I$, where I is an identity matrix of order 2×2. Then the value of $\alpha + \beta$ is equal to:

A) -10

B) -6

C) 6

D) 10

Answer: 10

Solution: Given $A=\begin{bmatrix}1 & 2\\ -2 & -5\end{bmatrix}$

So $A^2=\begin{bmatrix}1 & 2\\ -2 & -5\end{bmatrix}\begin{bmatrix}1 & 2\\ -2 & -5\end{bmatrix}=\begin{bmatrix}-3 & -8\\ 8 & 21\end{bmatrix}$

Also given $\alpha A^2+\beta A=2I$

i.e. $\alpha\begin{bmatrix}-3 & -8\\ 8 & 21\end{bmatrix}+\beta\begin{bmatrix}1 & 2\\ -2 & -5\end{bmatrix}=2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$-3\alpha+\beta=2\ \ \&\ -8\alpha+2\beta=0$

Solving both the equations, we get

$\alpha=2,\ \beta=8\Rightarrow\alpha+\beta=10$

Q.37. $(p\wedge r)\Leftrightarrow(p\wedge{\sim}q)$ which is equivalent to ${\sim}p$. Then r will be

A) p

B) ${\sim}p$

C) q

D) ${\sim}q$

Answer: q

Solution:

p	q	${\sim}p$	${\sim}q$	$p\wedge q$	$p\wedge{\sim}q$	$p\wedge q\Leftrightarrow p\wedge{\sim}q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

We can observe that $(p\wedge q)\Leftrightarrow(p\wedge{\sim}q)\equiv{\sim}p$

Hence, $r\equiv q$

Q.38. The remainder of $(2021)^{2022}+(2022)^{2021}$ when divided by 7 is

A) 0

B) 1

C) 2

D) 6

Answer: 0

Solution: We know $2023=7\times289$

So $(2021)^{2022}+(2022)^{2021}$

$=(2023-2)^{2022}+(2023-1)^{2021}$

$=7m_1+2^{2022}+7m_2+(-1)^{2021}$

$=7(m_1+m_2)+8^{674}-1$

$=7(m_1+m_2)+\left(7+1\right)^{674}-1$

$=7(m_1+m_2)+7m_3+1-1$

$=7(m_1+m_2+m_3)$

This is a multiple of 7

Hence, remainder is 0

Q.39. Let R_1 and R_2 be two relation defined on $\mathbb{R}$ by $aR_1b:ab\geq0$ and $aR_2b\Leftrightarrow b\geq a, a,b\in\mathbb{R}$ then the correct statement is:

A) Both R_1 and R_2 are equivalence relations

B) R_1 is equivalence but R_2 is not

C) R_2 is equivalence but R_1 is not

D) Both R_1 and R_2 are not equivalence relations

Answer: R_1 is equivalence but R_2 is not

Solution: R_1 is:

reflexive as $a^2 \geq 0$

symmetric as $ab \geq 0 \Rightarrow ba \geq 0$

tranistive as $ab \geq 0,\ bc \geq 0 \Rightarrow ac \geq 0$

Hence, R_1 is an equivalence relation.

R_2 is:

not reflexive as $b \geq a \not\Rightarrow a \geq b$

Hence, R_2 is not an equivalence relation.

Q.40. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ satisfies $\left(4\sqrt{\frac{2}{5}}, 3\right)$ and $e = \frac{1}{4}$, then $a^2 + b^2$ is equal to ______ .

Answer: 31

Solution: For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}}$

i.e. $\frac{1}{4} = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \frac{b^2}{a^2} = \frac{15}{16} \Rightarrow 16b^2 = 15a^2$

Since $\left(4\sqrt{\frac{2}{5}}, 3\right)$ lies on the ellipse

so $\frac{16 \cdot 2}{5a^2} + \frac{9}{b^2} = 1$

$\Rightarrow \frac{16 \cdot 2 \cdot 3}{16b^2} + \frac{9}{b^2} = 1$

$\Rightarrow \frac{15}{b^2} = 1 = b^2 = 15$

and $a^2 = 16$

$\therefore a^2 + b^2 = 31$

Q.41. The value of $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$ is equal to

A) -1

B) 1

C) $\frac{-1}{2}$

D) $\frac{1}{2}$

E) 0

Answer: $\frac{-1}{2}$

Solution: $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$

$=\frac{1}{2\sin\left(\frac{\pi}{7}\right)}\left\{2\cos\left(\frac{2\pi}{7}\right)\sin\left(\frac{\pi}{7}\right)+2\cos\left(\frac{4\pi}{7}\right)\sin\left(\frac{\pi}{7}\right)+2\cos\left(\frac{6\pi}{7}\right)\sin\left(\frac{\pi}{7}\right)\right\}$

$=\frac{1}{2\sin\frac{\pi}{7}}\left\{\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}+\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}+\sin\frac{7\pi}{7}-\sin\frac{5\pi}{7}\right\}$

Since, $2\cos A\sin B=\sin(A+B)-\sin(A-B)$

$=\frac{1}{2\sin\frac{\pi}{7}}\left\{-\sin\frac{\pi}{7}+\sin\pi\right\}$

$=\frac{-1}{2},\ (\because \sin\pi=0)$

Aliter:

Since, $e^{i\theta}=\cos\theta+i\sin\theta$

$\therefore \cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$

$=\operatorname{Re}\left\{e^{\frac{2\pi i}{7}}+e^{\frac{4\pi i}{7}}+e^{\frac{6\pi i}{7}}\right\}$

$Re\left\{\frac{\left[-1+\left(1+e^{2\pi i/7}+e^{4\pi i/7}+e^{6\pi i/7}+e^{-2\pi i/7}+e^{-4\pi i/7}+e^{-6\pi i/7}\right)\right]}{2}\right\}$

$=Re\left\{\frac{-1+(\text{ sum of seven roots of unity })}{2}\right\}$

$=-\frac{1+0}{2}=\frac{-1}{2}$

Q.42. Let $f(x)=a\sin\frac{\pi[x]}{2}+[2-x]$, where $[t]$ represents the greatest integer value less than or equal, then $\int_0^4 f(x)dx$ is equal to

A) -2

B) -1

C) 0

D) 2

Answer: -2

Solution: $\int_0^4 f(x)dx=\int_0^1(a(\sin 0)+1)dx+\int_1^2\left(a\left(\sin\frac{\pi}{2}\right)+0\right)dx+\int_2^3(a(\sin\pi)-1)dx+\int_3^4\left(a\left(\sin\frac{3\pi}{2}\right)-2\right)dx$

$=1+a-1-a-2=-2$

Q.43. $\int\frac{\left(x^2+1\right)e^x}{(x+1)^2}dx=f(x)e^x+C$, where C is a constant, then $\frac{d^3f}{dx^3}$ at $x=1$ is equal to

A) $\frac{3}{4}$

B) $\frac{3}{8}$

C) $-\frac{3}{2}$

D) $\frac{7}{8}$

Answer: $\frac{3}{4}$

Solution:

$\int \frac{\left(x^2+1\right)e^x dx}{(x+1)^2} = f(x)e^x + C$

$\int e^x \left(\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right)dx = f(x)e^x + C$

$\int e^x \left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right)dx = f(x)e^x + C$

We know that $\int e^x \left(f(x) + f'(x)\right) = e^x f(x) + c$

Here $f(x) = \frac{x-1}{x+1}$ & $f'(x) = \frac{2}{(x+1)^2}$

So $\int e^x \left(\left(\frac{x-1}{x+1}\right) + \frac{2}{\left(x+1\right)^2}\right)dx = f(x)e^x + C$

$\Rightarrow e^x \left(\frac{x-1}{x+1}\right) + C = e^x f(x) + C$

On comparing both sides we get $f(x) = \frac{x-1}{x+1}$

So $f'(x) = \frac{2}{(x+1)^2}$ & $f''(x) = \frac{-4}{(x+1)^3}$

$f'''(x) = \frac{12}{(x+1)^4}$,

Now $f'''(1) = \frac{12}{(1+1)^4} = \frac{12}{16} = \frac{3}{4}$

Q.44. If A is a 3×3 matrix with elements from $\{-1, 0, 1\}$, then the number of matrices A such that the sum of diagonal elements of AA^T is 6 is

A) ${}^9C_3(2)^6$

B) ${}^9C_2(2)^7$

C) ${}^9C_6(2)^3$

D) ${}^9C_3{}^9C_6$

Answer: ${}^9C_3(2)^6$

Solution:

Let $A = \begin{bmatrix} a & b & c \\ l & m & n \\ p & q & r \end{bmatrix}$

then $a^2 + b^2 + c^2 + l^2 + m^2 + n^2 + p^2 + q^2 + r^2 = 6$

Hence, total number of matrices will be ${}^9C_3(2)^6$ as only three places to be filled with 0 and six places with either of $-1/1$

Q.45. The number of points where tangents are vertical or horizontal to curve $y^5 + 9xy - 2x = 0$, are equal to ______ .

A) $0, \frac{5}{18}$

B) $0, \frac{5}{15}$

C) $0, \frac{4}{18}$

D) $0, \frac{4}{15}$

Answer: $0, \frac{5}{18}$

Solution: $y^5+9xy-2x=0$

Differentiating w.r.t x

$5y^4\frac{dy}{dx}+9x\frac{dy}{dx}+9y-2=0$

$\frac{dy}{dx}=\frac{2-9y}{5y^4+9x}$

For horizontal tangents, $\frac{dy}{dx}=0 \Rightarrow y=\frac{2}{9}$

$\frac{dy}{dx}=\frac{2-9y}{5y^4+9x}$

For vertical tangents, $\frac{dx}{dy}=0 \Rightarrow 5y^4+9x=0$

$\Rightarrow\ x=-\frac{5}{9}y^4$

$\Rightarrow\ y^5-5y^5+\frac{10}{9}y^4=0$

$\Rightarrow\ 4y^5=\frac{10}{9}y^4\quad \Rightarrow y=0,\frac{5}{18}$